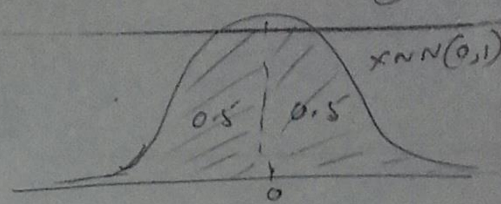
② ①

Tutorial 07.

Section A.



(1). X - The amount of drink

$$X \sim N(\mu, \sigma^2)$$
$$X \sim N(200, 15^2)$$

$\mu = 200$ ml/cup.
$\sigma = 15$ ml.

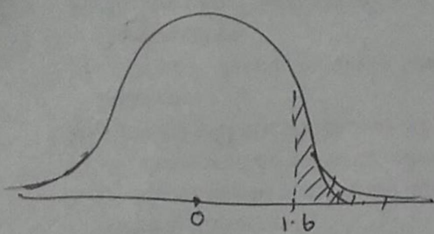
a). $P(X > 224) = P\left(\frac{X-\mu}{\sigma} > \frac{224-\mu}{\sigma}\right)$

$= P\left(Z > \frac{224-200}{15}\right)$

$= P(Z > 1.6)$

$= 1 - P(Z < 1.6)$

$= 1 - 0.9452 = 0.0548$



(b). $P(191 < X < 209)$

$\Rightarrow P\left(\frac{191-200}{15} < \frac{X-200}{15} < \frac{209-200}{15}\right)$

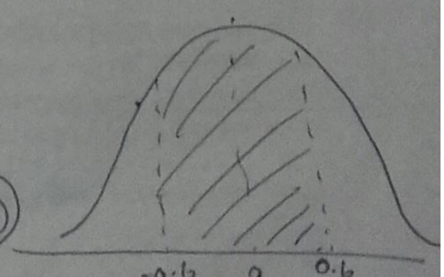
$\Rightarrow P(-0.6 < Z < 0.6)$

$= P(Z<0.6) - P(Z<-0.6)$

$= P(Z<0.6) - [1 - P(Z<0.6)]$

$= 2P(Z<0.6) - 1$

$= 2 \times 0.7257 - 1 = 0.4514$



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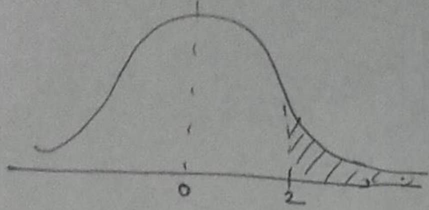
(c). $p(x>230) = p\left(z > \frac{230-200}{15}\right) = p(z>2)$.



$= 1 - p(z<2)$

$= 1 - 0.9772$

$= 0.0228$

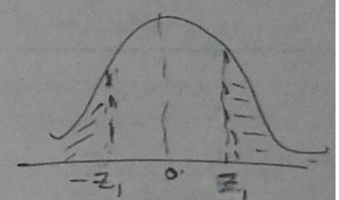
Hence, no. of cups that will overflow

$= 1000(0.0228)$

$= 22.8$

≈ 23 cups.

(d). $p(z < -z_1) = 0.25$.



~~$1 - p(z > z_1) = 0.25$~~

$\Rightarrow$

$1 - p(z < z_1) = 0.25$.

$0.75 = p(z < z_1)$

$z_1 = 0.67$

$\Rightarrow \quad -z_1 = -0.67$

$-z_1 = \frac{x-\mu}{\sigma} = \frac{x-200}{15} = -0.67$.

$x = 189.95$ ml.

Section B.

A second order linear differential eq^n has the form

$$P(x)\frac{d^2y}{dx^2} + Q(x)\frac{dy}{dx} + R(x)y = G(x) \longrightarrow ①$$

where P, Q, R & G are continuous f^{ns}.

If $G(x) = 0$ then ① is a homogeneonse linear eq^n. & If $G(x) \neq 0$ for some x, eq^n ① is Non homogeneons.

~~Case 1~~ If y_1 & y_2 are linearly independent sol^{ns} of $P(x)y'' + Q(x)y' + R(x)y$, and $P(x)$ is never zero $(P(x) \neq 0)$, then the general sol^n is given by $y(x) = c_1 y_1(x) + c_2 y_2(x)$

c_1 & c_2 are arbitrary constants.

$ar^2 + br + c = 0$ is called the <u>Auxiliary eq^n</u> (or characheristic eq^n) of the differential eq^n $ay'' + by' + cy = 0$.

Case I $b^2-4ac>0$

If the roots r_1 & r_2 of the Auxiliary eq^n $ar^2+br+c=0$ are real & unequal, then general sol^n of $ay''+by'+cy=0$ is $y=c_1e^{r_1x}+c_2e^{r_2x}$.

(i) $y''+y'-6y=0$.

Auxilary eq^n is $r^2+r-6=0$.

$\Rightarrow (r-2)(r+3)=0$

$\Rightarrow r=2$ or $r=-3$.

G.S. is $y=c_1e^{2x}+c_2e^{-3x}$. c_1 & c_2 are arbitrary constants.

Case II $b^2-4ac=0$

If the auxilary eq^n $ar^2+br+c=0$ has only one real root r, then the general sol^n of $ay''+by'+cy=0$ is $y=c_1e^{rx}+c_2xe^{rx}$.

exa: Solve $4y''+12y'+9y=0$

Auxilary eq^n is $4r^2+12r+9=0$

$\Rightarrow (2r+3)^2=0$.

$\Rightarrow r=-3/2$

G.S. is $y=c_1e^{-3x/2}+c_2xe^{-3x/2}$

c_1 & c_2 are arbitrary constants.

Case III $b^2 - 4ac < 0$

If roots of A.E. $ar^2 + br + c = 0$ are complex numbers $r_1 = \alpha + i\beta$, $r_2 = \alpha - i\beta$, then the G.S. of $ay'' + by' + cy = 0$ is

$$y = e^{\alpha x}(c_1 \cos\beta x + c_2 \sin\beta x).$$

Exa: Solve $y'' - 6y' + 13y = 0$

A.E. $r^2 - 6r + 13 = 0$

$$\Rightarrow r = \frac{6 \pm \sqrt{-16}}{2} = 3 \pm 2i$$

G.S. is $y = e^{3x}(c_1 \cos 2x + c_2 \sin 2x)$

c_1 & c_2 are arbitrary constants

Tutorial 7

Section C (Numerical Methods)

I. Fixed Point method

Let $f(x) = x^5 - x - 1$

$f(x) = 0$

$x^5 - x - 1 = 0$

$x^5 = x + 1$

$x = (x+1)^{1/5} = g(x)$

$x_{k+1} = g(x_k) \leftarrow$ Iterative Formula

Conditions

(i). Let choose the interval $[0,2]$

$\forall\, x \in [0,2],\; g(x) \in [0,2]$

Here $g(0) = 1 \in [0,2]$

$g(2) = 1.245 \in [0,2]$

(ii). $g'(x) = \frac{1}{5}(x+1)^{-4/5} = \frac{1}{5(x+1)^{4/5}}$

$g'(x)$ exists on $(0,2)$ with the property that $\exists$ a positive constant $0 < r < 1$ s.t

$|g'(x)| \leq r \quad (r = 0.5) \quad \forall\, x \in (0,2)$

choose $x_0 = 0.5 \in [0,2]$

$x_1 = g(x_0) = 1.0845$

$x_2 = g(x_1) = 1.1582$

$x_3 = g(x_2) = 1.1663$

$x_4 = g(x_3) = 1.1672$

$x_5 = g(x_4) = 1.1673$

$x_6 = g(x_5) = 1.1673$

$x_7 = g(x_6) = 1.1673$