Tutorial 06

Section A

1. Let X = Number of floppy discs which work.

Applying the Binomial distribution

$$P(X=r) = {}^nC_r\, p^r (1-p)^{n-r}.$$

$p = 0.95$; $n = 3$
$1-p = 0.05$

$\therefore\ X \sim B(n, p) \Rightarrow X \sim B(3, 0.95).$

(a) $P(X=0) = {}^3C_0\, p^0 (1-p)^3$
$= 1 \times 1 \times (0.05)^3 = 0.000125$

(b). $P(X=1) = {}^3C_1 (0.95)^1 (0.05)^2$
$= 3 \times 0.95 \times 0.05^2 = 0.007125$

(c) $P(X=2) = {}^3C_2 (0.95)^2 (0.05)^1$
$= 0.135375.$

(d). $P(X=3) = {}^3C_3 (0.95)^3 (0.05)^0 = (0.95)^3$
$P(X=3) = 0.857375$

(e) Now $X \sim B(3, 0.95)$

$E(X) = np$

$= 3 \times 0.95 = 2.85$

$V(X) = E(X^2) - [E(X)]^2$

$= np(np + q) - (np)^2$

$V(X) = npq$

$= 3 \times 0.95 \times 0.05$

$= 0.1425$

or

02. Let X be the number of circuits which work successfully.

$\therefore X \sim B(30, 0.9)$

(a) $P(X=30) = {}^{30}C_{30}(0.9)^{30}(0.1)^{0} = 0.04239$ //

(b) $P(X \geq 28) = P(X=28) + P(X=29) + P(X=30)$

$= {}^{30}C_{28}(0.9)^{28}(0.1)^{2} + {}^{30}C_{29}(0.9)^{29}(0.1)^{1}$

$+ {}^{30}C_{30}(0.9)^{30}(0.1)^{0}$

$= 0.41135$ //

(4). Let x be the number of companies to which the engineer is called.

(a). $P(x=4) = (0.1)^4 = 0.0001.$

$\{\text{Here } x \sim B(4, 0.1)\}$.

(b). $P(x \geq 3) = P(x=3) + P(x=4).$

$= {}^4C_3 (0.1)^3 (0.9)^1 + (0.1)^4$

$= 0.0037$ //

(c). $P(x=4 \mid x \geq 1) = \dfrac{P(x=4 \text{ \& } x \geq 1)}{P(x \geq 1)}$

$= \dfrac{P(x=4)}{P(x \geq 1)}$

$= \dfrac{P(x=4)}{1 - P(x=0)}$

$= \dfrac{(0.1)^4}{1-(0.9)^4}$

$= 0.00029$ //

(d). Let A denote the event that he is called to company A.

(d) $P(X=4 \mid A) = \dfrac{P(X=4 \ \& \ A)}{P(A)}$

$= \dfrac{P(X=4)}{P(A)} = 0.001$ //

5. Let X be the number of vehicles arriving at given time interval. ~~(40 seconds)~~.

In 40 seconds we expect 4 vechicles.

per hour $\longrightarrow$ 360,

per second $\longrightarrow \dfrac{360}{3600}$

for 40 seconds $\longrightarrow$ 4

$\therefore \lambda = 4$

Now $X \sim P(\lambda)$

$P(X=x) = \dfrac{e^{-\lambda}\lambda^{x}}{x!}$

(a). $P(X=5) = \dfrac{e^{-4}4^{5}}{5!} = 0.15629$ //

(b). $P(X<5) = P(X=0)+P(X=1)+P(X=2)+P(X=3)+P(X=4)$

$= e^{-\lambda}\left[1 + \lambda + \dfrac{\lambda^2}{2!} + \dfrac{\lambda^3}{3!} + \dfrac{\lambda^4}{4!}\right]$

$= 0.628$

(5) vechicles will not cleared if more than 5 are waiting.

$$\cancel{P(x>5) = 1 - P(x=5) - P(x<5)}$$

$$P(x>5) = 1 - P(x \leq 5)$$

$$= 1 - \{P(x=5) + P(x<5)\}.$$

$$=$$

Tutorial 6

Section C (Numerical Methods)

1. $f[x_0, x_1] = \dfrac{f(x_1)-f(x_0)}{x_1 - x_0}$

$f[x_1, x_2] = \dfrac{f(x_2)-f(x_1)}{x_2 - x_1}$

$f[x_0, x_1, x_2] = \dfrac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0}$

The polynomial given for the Newton divided differences method is;

$p(x) = f(x_0) + f[x_0, x_1](x-x_0) + f[x_0, x_1, x_2](x-x_0)(x-x_1)$

Let it to be interpolated by $p_2(x)$

$p(x) = p_2(x) + E$; E is the error term

$= f(x_0)\dfrac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} + f(x_1)\dfrac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} + f(x_2)\dfrac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$

$+ E$

Error of the Lagrange interpolation polynomial $= \dfrac{f^{(n+1)}(\xi)}{(n+1)!} W(x)$; $\xi \in (x_0, x_2)$

$= \dfrac{f^{(3)}(\xi)}{3!} W(x) = \dfrac{p^{(3)}(\xi)}{3!} W(x)$

Since

$p(x) = f(x_0) + f[x_0, x_1](x-x_0) + f[x_0, x_1, x_2](x-x_0)(x-x_1)$

$p^{(1)}(x) = f[x_0, x_1] + f[x_0, x_1, x_2]\left[(x-x_1)+(x-x_0)\right]$

$p^{(2)}(x) = 2f[x_0, x_1, x_2]$

$p^{(3)}(x) = 0$

$\therefore E = 0$

$\therefore$ Both polynomials are same.

Newton's Divided difference Method

x_i	$f[\,]=f(\,)$	$f[\,,\,]$	$f[\,,\,,\,]$	$f[\,,\,,\,,\,]$
1	1 $f[x_0]$			
		0 $f[x_0,x_1]$		
2	1 $f[x_1]$		$1/2$ $f[x_0,x_1,x_2]$	
		1 $f[x_1,x_2]$		$-1/12$ $f[x_0,x_1,x_2,x_3]$
3	2 $f[x_2]$		$1/6$ $f[x_1,x_2,x_3]$	
		$3/2$ $f[x_2,x_3]$		
5	5 $f[x_3]$			

$$P(x) = 1 + 0\cdot(x-1) + \frac{1}{2}(x-1)(x-2) - \frac{1}{12}(x-1)(x-2)(x-3)$$

$$= 1 + \frac{1}{2}(x^2-3x+2) - \frac{1}{12}(x-3)(x^2-3x+2)$$

$$= 1 + \frac{1}{2}(x^2-3x+2) - \frac{1}{12}(x^3-3x^2+2x-3x^2+9x-6)$$

$$= -\frac{1}{12}x^3 + x^2 - \frac{29}{12}x + \frac{5}{2}$$