

**UNIVERSITY OF MORATUWA**

MSC/POSTGRADUATE DIPLOMA IN FINANCIAL MATHEMATICS

MA 5100 INTRODUCTION TO STATISTICS**THREE HOURS****September 2008**Answer **FIVE** questions and **NO MORE**.

Question 1

(a) In finance, an efficient market is defined as one that allocates funds to the most productive use. Business Week recently surveyed 110 financial analysts who work for private manufacturing firms in the effort to sell their firms' securities, 42 felt markets were efficient, while 31 of 75 analysts who work for brokerage houses assisting in these sales agreed that markets were efficient. Test whether there appears to be a difference in the proportion of these two types of analysts who accept the concept of market efficiency at 5% level of significance.

(b) As a marketing specialist for Federated department stores, you have been asked by your supervisor to prepare a 95 percent confidence interval for the difference in the mean revenues of stores in the South and those in the Midwest. In this effort, 75 stores in the South that you select at random report mean daily revenues of \$1,125 with a standard deviation of \$213, while 83 stores in the Midwest report daily mean revenues of \$1,282 with a standard deviation of \$298. Produce 95% confidence interval and the proper interpretation.

Question 2

(a) The residents of a small town are worried about a rise in housing costs in the area. The mayor thinks that home prices fluctuate with land values. Data on 10 recently sold homes and the cost of the land on which they were built are shown in Table 1 in thousands of dollars. Identify the dependent and the independent variable. Find correlation between land value and building a

house. Construct and interpret the regression model. On this basis, does it appear that the mayor is correct?

TABLE 1

Land Values	7	6.9	5.5	3.7	5.9	3.8	8.9	9.6	9.9	10
Cost of the House	67	63	60	54	58	36	76	87	89	92

(b) The results of a study by the American Marketing Association to determine the relationship between the importance store owners attach to advertising and the size of store they own are shown in Table 2. Would it seem that all store owners place the same emphasis on advertising? Set $\alpha = 0.05$. State the hypotheses H_0 and H_1 .

TABLE 2

Size	Advertising		
	Important	Not Important	No Opinion
Small	20	52	32
Medium	53	47	28
Large	67	32	25

Question 3

(a) Smile Bright sells toothpaste in 17 grams containers. Management expects overfills and underfills to be random. If they are not, management assumes something is wrong with the fill system, and shuts down the production line. Should the line be shut down if containers measured 16.8, 18.2, 17.3, 17.5, 16.3, 17.4, 16.1, 16.9, 17, 18.1, 17.3, 16.2, 17.3, and 16.8 grams? [Hint: Take $\alpha = 0.05$]

(b) Records kept by the American Association show that only 30 percent males get an annual medical exam. Dr. Hart, the local cardiologist, consults the records of seven of his patients. What is the probability he will find

- (i) all seven has exams?
- (ii) none have exams?
- (iii) between one and three inclusive have exams?

Question 4

A particular company produces electrical components utilizing three non-overlapping work shifts. It is observed that 50%, 30% and 20% of the components are produced during shift 1, 2 and 3 respectively. Furthermore 6%, 10% and 8% components produced in shift 1, 2 and 3 respectively are defective. Determine

- (a) What percentage of all components is defective?
- (b) Given that a defective component is found, what is the probability that it was produced during shift 3?
- (c) If one shift has to be closed down which one should it be? What effect would it have on the overall defects to good products ratio?

Question 5

(a) A new integrated computer system is to be installed worldwide for a major corporation. Bids on this project are being solicited, and the contract will be awarded to one of the bidders. As a part of the proposal for this project, bidders must specify how long the project will take. There will be a significant penalty for finishing late. One potential contractor determines that the average time to complete a project of this type is 40 weeks with a standard deviation of 5 weeks. The time required to complete this project is assumed to be normally distributed.

- (i) If the due date of this project is set at 40 weeks, what is the probability that the contractor will have to pay a penalty (i.e., the project will not be finished on schedule)?
- (ii) If the bidder wishes to set the due date in the proposal so that there is only a 5% chance of being late (and consequently only a 5% chance of having to pay a penalty), what due date should be set?

(b) Suppose that a radio contains six transistors, two of which are defective. Three transistors are selected at random, removed from the radio, and inspected. Let Y equal the number of defectives observed, where $Y = 0, 1$, or 2 . Find the probability distribution for Y . Express your results graphically as a probability histogram.

Question 6

(a) From a group of three Republicans, two Democrats, and one Independent, a committee of two people is to be randomly selected. Let Y_1 denote the number of Republicans and Y_2 the number of Democrats on the committee. Show that joint probabilities of Y_1 and Y_2 could be computed as in Table 3.

Table 3

		Y_2			
Y_1		0	1	2	Total
	0	0	3/15	3/15	6/15
	1	2/15	6/15	0	8/15
	2	1/15	0	0	1/15
Total		3/15	9/15	3/15	1

- (i) Find marginal distribution of Y_1 and Y_2 .
- (ii) Find covariance between Y_1 and Y_2 .
- (iii) Find the probability that $p(Y_1 \leq 1)$.

(b) Eleven school boys were given a test drawing without any tuition and after months' tuition. A test of equal difficulty was held at both occasions. Do the marks as shown in Table 4, give evidence that the students have benefited by extra coaching?

TABLE 4

Boys	1	2	3	4	5	6	7	8	9	10	11
Marks Test 1	23	20	19	21	18	20	18	17	23	16	19
Marks Test 2	24	19	22	18	20	22	20	20	23	20	17

Question 7**TABLE 5**

Outcome	New drilling operation		Existing operation	
	Probability	Expected revenue (Rs. million)	Probability	Expected revenue (Rs. million)
Success	0.75	800	0.85	700
Failure	0.25	200	0.15	250

The Oil India Corporation (OIC) is considering whether to go for an offshore oil drilling contract to be awarded in Sri Lanka. If the bid value would be Rs. 600 million with a 65% chance of gaining the contract, they may set up a new drilling operation or move already existing operation which has proved successful, to the new site. The probability of success and expected returns are as shown in Table 5;

If the corporation do not bid or lose the contract, they can use the Rs. 600 million to modernize their operation. This would result in a return of either 5% or 8% on the sum invested with probabilities of 0.45 and 0.55. Assume that all costs and revenues have been discounted to the present value.

- (i) Construct a decision tree for the problem showing clearly the course of action.
- (ii) By applying an appropriate decision criteria recommend whether or not the Oil India Corporation should bid the contract.
- (iii) What would be the financial return if they bid?

Question 8

(a) The manufacture of colour television sets offers one year warranty of free replacement if the picture tube fails. He estimates the time to failure T is years to be a random variable with the following probability density function

$$f(t) = \frac{1}{4} e^{-\frac{1}{4}t} \quad t > 0$$

- (i) What percentage of the sets will we have to service?
- (ii) If the profit per sale is Rs, 2000 and the replacement of a picture tube costs Rs, 200, find the expected profit per sale.

(b) Suppose that a unit of mineral ore contains a proportion Y_1 of metal A and a proportion Y_2 of metal B. Experience has shown that the joint probability density function of (y_1, y_2) is uniform over the region $0 \leq y_1 \leq 1$, $0 \leq y_2 \leq 1$, $0 \leq y_1 + y_2 \leq 1$. Let $U = Y_1 + Y_2$, the proportion of metals A and B per unit.

- (i) Find the probability density function for U .
- (ii) Find $E(U)$ by using the answer to part (a).
- (iii) Find $E(U)$ by using only the marginal densities of Y_1 and Y_2 .