UNIVERSITY OF MORATUWA

MSC/POSTGRADUATE DIPLOMA IN FINANCIAL MATHEMATICS 2009

MA 5100 INTRODUCTION TO STATISTICS

THREE HOURS **November 2009**

Answer **FIVE** questions and **NO MORE**.

Question 1

(a) A supplier of kerosene has a weekly demand Y possessing a probability density function given by

$$f(y) = \begin{cases} y, & 0 \le y \le 1 \\ 1, & 1 \le y \le 1.5 \\ 0, & \text{elesewhere} \end{cases}$$

with measurements in hundreds of gallons. The supplier's profit is given by U = 10Y- 4.

(i) Find the probability density function for U. (ii) Use the answer to (i) to find E(U).

(b) The life time X of certain electronic component is distributed as normal with mean 23 hours and standard deviation 2 hours . Replacing such a component in the course of certain job causes expensively delay, so a new component is fitted before starting the job. If the component lasts for x hours, it ensures a profit of Rs. p where,

$$p = \begin{cases} -100 & 0 < x < 20 \\ 20 & 20 < x < 25 \\ 40 & x \ge 25 \end{cases}$$

Find expected profit.

Question 2

(a)Environmental Authority set a maximum noise level for heavy trucks at 83 decibels (Environment News, October 2008). How this limit is applied will greatly affect the industry and the public. One way to apply the limit would be to require all trucks to conform to the noise limit. A second but less satisfactory method would be to require the truck fleet mean noise level to be less than the limit. If the latter were the rule, variation in the noise level from truck to truck would be important because a large value of σ^2 would imply many trucks exceeding the limit, even if the mean fleet level was 83 decibels. A random sample of six heavy trucks in 2008 produced the following noise levels (in decibels):

85.4, 86.8, 86.1, 85.3, 84.8, 86.0. Use these data to construct a 95 % confidence interval for σ^2, the variance of the truck noise emission readings. Interpret your results.

(b) In a bolt factory, there are four machines *A*, *B*, *C*, *D* manufacturing 20%, 15%, 25% and 40% of the total output respectively. Of their outputs 5%, 4%, 3% and 2% in the same order are defective bolts. A bolt is chosen at random from the factory's production and is found defective. What is the probability that the bolt was manufactured by machine *A* or machine *D?*

Question 3

The logarithmic distribution with parameter $\theta (0 < \theta < 1)$, has discrete probability density function

$$f(x) = \frac{k\theta^x}{x} \quad x = 1, 2, 3, \ldots\ldots$$

(i) Find the constant k as function of θ

(ii) Show that the mean of this distribution is $\frac{k\theta}{1-\theta}$

(iii)By finding $E(x(x-1)$ or otherwise, show that $E(x^2) = \frac{k\theta}{(1-\theta)^2}$

Question 4

A studies of the habits of white-tailed deer that indicate that they live and feed within very limited ranges, approximately 150 to 205 acres. To determine whether there was a difference in the ranges of deer located in two different geographical areas, forty deer were caught, tagged, and fitted with small radio transmitters. Several months later, the deer were tracked and identified and the distance y from the release point was recorded. The mean and standard deviation of the distances from the release point were as follows:

	Location	
	1	2
Sample size	40	40
Sample mean	2980 ft	3205 ft
Sample standard deviation	1140 ft	963 ft
Population mean	11	42

(a) If you have no preconceived reason for believing one population mean to be larger than another, what would you choose for your alternative hypothesis? Your null hypothesis?

(b)Would your alternative hypothesis in part (a) imply a one- or a two-tailed test? Explain.

(c) Do the data provide sufficient evidence to indicate that the mean distances differ for the two geographical locations? Test using $\alpha = 5\%$.

Question 5

A city is considering replacing its fleet of municipally owned, gasoline-powered automobiles by electric cars. The manufacturer of the electric cars claims that the city will experience significant savings over the life of the fleet if it converts, but the city has doubts. If the manufacturer is correct, the city will save 1 million dollars. If the new technology is faulty, as some critics suggest, the conversion will cost the city $450000. A third possibility is that neither situation will occur and the city will break even with conversion. According to the recently completed consultant’s report , the perspective probabilities of these three events are 0.25, 0.45 and 0.30. The city has before it a pilot program that if implemented would indicate the potential cost or savings in a conversion to electric cars. The program involves renting three electric cars for 3 months and running them under normal conditions. The cost to the city of this pilot program would be $50000. The city's consultant believes that the results of the pilot program would be significant but not conclusive; she submits Table 1, a compilation of probabilities based on the experience of other cities, to support her contention. What actions should the city take if it wants to maximize expected savings?

Table 1

	Savings	No change	A Loss
Saves money	0.6	0.3	0.1
Breaks even	0.4	0.4	0.2
Losses money	0.1	0.5	0.4

Question 6

Let Y_1 and Y_2 have the joint probability density function given by

$$f(y_1, y_2) = \begin{cases} ky_1 y_2, & 0 \le y_1 \le 1,\ 0 \le y_2 \le 1, \\ 0, & \text{elsewhere} \end{cases}$$

(i) Find the value of K that makes this a probability density function.

(ii) Find the joint distribution function for Y_1 and Y_2.

(iii)Find $p(Y_1 \le 1/2,\ Y_2 \le 3/4)$.

(iv) Find the marginal probability density function of y_1.

Question 7

(a)A criminologist conducted a survey to determine whether the incidence of certain types of crime varied from one part of a large city to another. The particular crimes of interest were assault, burglary, larceny, and homicide. The following table shows the numbers of crimes committed in four areas of the city during the past year.

Type of Crime

District	Assault	Burglary	Larceny	Homicide
1	162	118	451	18
2	310	196	996	25
3	258	193	458	10
4	280	175	390	19

Can we conclude from these data at the 0.05 level of significance that the occurrence of these types of crime is dependent upon the city district?

(b)In finance, an efficient market is defined as one that allocates funds to the most productive use. Business Week recently surveyed 110 financial analysts who work for private manufacturing firms in the effort to sell their firms' securities, 42 felt markets were efficient, while 31 of 75 analysts who work for brokerage houses assisting in these sales agreed that markets were efficient. Test whether there appears to be a difference in the proportion of these two types of anlaysts who accept the concept of market efficiency at 5% level of significance.

Question 8

(a) The amount of gas required to heat a home depends on the out-door temperature. A study recorded the average daily gas consumption of a house y (in hundreds of cubic feet) for each month during one heating season. The explanatory variable x is the average number of heating degree days per day during the month. (One heating degree day is accumulated for each degree a day's average temperature is below 650^0 F). The: data are

	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May	Jun
X	15.6	26.8	37.8	36.4	35.5	18.6	15.3	7.9	0.0
Y	5.2	6.1	8.7	8.5	8.8	4.9	4.5	2.5	1.1

The summary data are

$$\sum x_t = 193.9,\ \sum y_t = 50.3,\ \sum x_t^2 = 5618.11,\ \sum y_t^2 = 341.35,\ \sum x_t y_t = 1375.00$$

It is thought that linear relationship exist between y and x of the form

$y_t = \alpha + \beta x_t + \in$ where $\in_t \sim N(0, \sigma^2)$

(i) Plot the data to confirm that there is a linear relationship between the two variable.

(ii) Explain the meaning of the slope (β) and (α) parameters.

{iii}Calculate the least squares estimates of α and β, and include the estimated line on your plot.

(iv) Calculate the estimated error variance.

(v) Calculate a 95% confidence interval for β .

(b) In a study on the relationship between the height (u) and weight (v) of n species of monkeys a researcher collected data from 10 adult individuals. The summary statistics were CS(u, u) = 190.7, CS(v, v) =270.3 and CS(u,v) =1 57.6. Calculate the sample correlation coefficient between height and weight and test whether the population correlation coefficient is zero.