1. Some Equations from Physics

Bournoulli's Equation $P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$
Simple Harmonic Motion $\ddot{x} = -\omega^2 x$
Velocity in Polar coordinates $\dot{\boldsymbol{r}} = \dot{r}\boldsymbol{e_r} + r\dot{\theta}\boldsymbol{e_\theta}$
Acceleration in Polar coordinates $\ddot{\boldsymbol{r}} = (\ddot{r} - r\dot{\theta}^2)\boldsymbol{e_r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\boldsymbol{e_\theta}$
Resistor $V_R = IR$
Inductor $V_L = L\frac{dI}{dt}$
Capacitor $V_C = \frac{Q}{C}$ or $\frac{dV_C}{dt} = \frac{1}{C}\frac{dI}{dt}$
Kichhoff's Law(for a LRC circuit) $V_S = V_R + V_L + V_C$

2. Some Equations from Mathematics

Elliptic integral of the first kind $F(k) = \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1-k^2\cos^2\theta}}$
Second order linear ordinary differential equation with constant coefficients
$f(x) = y'' + by' + cy = \frac{d}{dx}(\frac{dy}{dx} - \alpha) - \beta(\frac{dy}{dx} - \alpha)$
Solution as a two first order linear ODEs $f(x) = \frac{dz}{dx} - \beta z, z = \frac{dy}{dx} - \alpha$
Trapeziodal Rule $\int_a^b f(x)dx \approx \frac{h}{2}\left(f(a) + 2\sum_{k=1}^{n-1} f(a+kh) + f(b)\right), h = \frac{b-a}{n}$
Polar coordinates $\boldsymbol{r} = r\cos\theta\boldsymbol{i} + r\sin\theta\boldsymbol{j} = r\boldsymbol{e_r}, \boldsymbol{e_\theta} = -\sin\theta\boldsymbol{i} + \cos\theta\boldsymbol{j}$
Euler's formula $e^{i\theta} = \cos\theta + i\sin\theta, i^2 = -1$
Complex number $z = x + iy = re^{i\theta}, r = |z|, \theta = \text{Arg}z$

Example 1.

1. Show that the sum of the distance to the perimeter from the two foci of an ellipse is a constant.
2. Show that an Ellipse can be written $r = \frac{p}{1+e\cos\theta}$*,p=semi-latus rectum, e=eccentricity*
3. A cylindrical container of radius R *and height* H *has a hole of radius* r *at the bottom. How long will it take for the container full of water to drain?*
4. How long will it take if the container is the parabola $y = x^2, x \in [0,1]$ *rotated about the* y *axis*
5. Solve the ODE $y'' + 5y' + 6y = \sin x$
6. Find the period of oscillation of a pendulum of length ℓ *which is oscillating at an angle* 2α*.*
7. Deduce the period of oscillation when $\alpha \to 0$
8. Write differential equations for planetary motion using Newton's laws and show the Kepler's laws of planetary motion:
The orbit of every planet is an ellipse with the Sun at one of the two foci.
A line joining a planet and the Sun sweeps out equal areas during equal intervals of time.
The square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of its orbit.
9. Without directly using calculus, try to show that the ratio circumference/radius of a circle is a constant. How to show that angle is independent of radius?
10. Write down rational approximations to π *using its Continued Fraction.*