UNIVERSITY OF MORATUWA

MSC/POSTGRADUATE DIPLOMA IN FINANCIAL MATHEMATICS

MA 5102 OPERATIONAL RESEARCH TECHNIQUE I

THREE HOURS **November 2009**

Answer **FIVE** questions and **NO MORE**.

Question 1

A company manufactures products A, B, C and D which are processed by planner, milling, drilling and assembly departments. The requirements per unit of product in hours and contribution are as follows:

Table 1

Department	Planner	Milling	Drilling	Assembly	Contribution/unit
Product A	0.5	2.0	0.5	3.0	Rs. 8
Product B	1.0	1.0	0.5	1.0	Rs. 9
Product C	1.0	1.0	1.0	2.0	Rs. 7
Product D	0.5	1.0	1.0	3.0	Rs. 6

Capacities of various departments and minimum sales requirements are

Table 2

Department	Capacity (hours)	Minimum Sales requirements
Planner	1,800	Product A 100 units
Milling	2,800	Product B 600 units
Drilling	3,000	Product C 500 units
Assembly	6,000	Product D 400 units

(a)Formulate as linear programmming problem so that the number of products A, B, C and D to be manufactured to maximize production. (b) Determine the total maximum contribution for products A, B, C and D.

(c) Determine the slack time in each department.

Question 2

A firm that makes three products, and has three machines available as resources, constructs the following LP problem:

Optimis profit $Z = 4X_1 + 4X_2 + 7X_3$

Subject to:

$X_1 + 7X_2 + 4X_1 \leq 100$ (hours of machine 1)
$2X_1 + X_2 + 7X_1 \leq 110$ (hours of machine 2)
$8X_1 + 4X_2 + X_1 \leq 100$ (hours of machine 3)

Solve this problem and answer these questions:

(a) Before the third iteration of the simplex method, which machine still has unused time available?

(b) When the final solution is reached, is there any unused time available on any of the three machines?

(c) What would it be worth to the firm to make an additional hour of time available on the third machine?

(d) How much would the firm's profit increase if an extra 10 hours of time were made available on the second machine at not extra cost?

Question 3

An Electricity Generating Board (B) wishes to set up a generating project which is opposed by a group of Protesters (P). The two parties B and G have 3 and 4 strategies respectively to achieve their own goals. A think tank team called by B scored marks from 0 to 9, that P will get for a pair of strategies adopted by the two parties, as shown below:-

	P_1	P_2	P_3	P_4
B_1	3	1	-1	-3
B_2	-1	-2	3	5
B_3	4	3	-3	6

[For instance if B uses B_1 and P uses P_2 then P will gain 1.]

Examine using principles of game theory whether the problem has a unique saddle point based on pure strategies.

Convert the above problem to a mixed strategy problem in which B attempts to minimize gain of P.

Also show how to solve the latter problem easily.

(b) The following pay off matrix shows non zero sum game of players 1and 2. Discuss Nash equilibria of the game.

	B_1	B_2	B_3
A_1	(2,1)	(2,1)	(0,3)
A_2	(2,1)	(0,2)	(2,1)
A_3	(1,2)	(2,1)	(2,0)

Question 4

Consider the following table summarizing the details of a project:

Table 3

Activity	Predecessor(s)	Duration (weeks)		
		a	m	b
A	__	4	4	10
B	__	1	2	9
C	__	2	5	14
D	A	1	4	7
E	A	1	2	3
F	A	1	5	9
G	B, C	1	2	9
H	C	4	4	4
1	D	2	2	8
J	E, G	6	7	8
K	F, H	2	2	8
L	F, H	5	5	5
M	1, J, K	1	2	9
N	L	6	7	8

(a) Construct the project network.

(b) Find the expected duration and variance of each activity.

(c) Find the critical path and the expected project completion time.

(d) What is the probability of completing the project on or before 35 weeks'?

(e) If the probability of completing the project is 0.85, find the expected project completion time.

Question 5

Consider the following game tree. At a leaf, the top payoffs are for player I, the bottom payoffs are for player II.

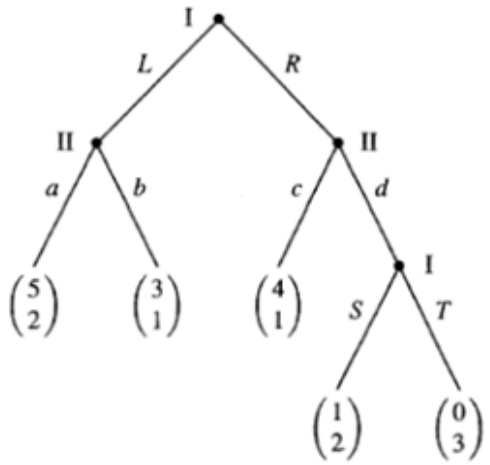

Figure 1

(a) What is the number of strategies of player I and of player II?

(b) How many reduced strategies do they have?

(c) Give the reduced strategic form of the game.

(d) What are the Nash equilibria of the game in reduced strategies?

(e) What are the subgame perfect Nash equilibria of the game?

Question 6

A perfume company products four types of perfume p_1, p_2, p_3, p_4 using three kinds of raw material R_1, R_2, R_3. The amounts of raw material used to produce 1 gm of each perfume and the resulting profit are follows:

Table 4

	R_1	R_2	R_3	Profit($/gm)
P_1	3	1	4	19
P_2	2	1	3	13
P_3	1	1	3	12
P_4	2	1	4	17

The company has 225units of R_1, 117 units of R2, and 420 units of R_3 available. Find how much of each perfume should be produced to maximize profit.

Formulating and solving the problem using the simplex Algorithm we obtain:

Find $x_1, x_2, x_3, x_4 \in R$ to maximize $z = 19x_1 + 13x_2 + 12x_3 + 17x_4$ subject to

$$3x_1 + 2x_2 + x_3 + 2x_4 \le 225$$
$$x_1 + x_2 + x_3 + x_4 \le 117$$
$$4x_1 + 3x_2 + 3x_3 + 4x_4 \le 420$$

And $x_i \ge 0$ *for* $0 \le i \le 4$.

When we solve this problem using the simplex Algorithm we obtain the final Optimal Table, dictionary;

Table 5 Optimal Simplex Tableau

	x_1	x_2	x_3	x_4	s_1	s_2	s_3	rhs
Z	0	1	0	0	2	1	0	1827
x_1	1	1	0	0	1	2	-1	39
x_3	0	1	1	0	0	4	-1	48
x_4	0	-1	0	1	-1	-5	2	30

Solve the following independent variations of this problem.

(a) The profit from p_2 increases from \$ 13 to \$ 15.

(b) The amount of R_2 available increases from 117 units to 125 units.

(c) The company develops a new perfume P_5 which requires 3 units of R_1, one unit of R_2, 2 units of R_2, 2 units of R_3 and gives \$14 profit.

(d) The amount of P_3 produced can be at most five times the amount of p_2.

(e) The profit from P_1changes from \$ 19 to. \$ (19 + p). Find the optimal Solution for all $-1 \le p \le 4$.

(f) The company can buy extra units of R_1at a cost of \$q / unit. For what values of q should it do this? Assuming q = 1, how many units of R_1 should it buy and what is the new maximum profit?

Question 7

A trip from Colombo to Kataragama takes six hours by bus. A typical table of the bus service in both directions is given below.

Table 6 Time table

Departure from Colombo	Colombo to Kataragama line or route number	A rival at Kataragama
06.30	a	12.30
08.30	b	14.30
11.30	c	17.30
19.00	d	01.00
00.30	e	06.30

Table 7

arrival at Colombo	Kataragama to Colombo line or route number	Departure from Kataragama
11.3	1	5.30
15.00	2	9.00
21.00	3	15.00
00.30	4	18.30
06.00	5	00.00

The cost of providing this service by the transport company depends upon the time spent by the bus crew (driver and conductor) away from their Places in addition to service times. There are five crew. There is a constraint that every crew should be provided with more than 4 hours of rest before the return trip again and should not wait for more than 24 hours for the return trip. The company has residential facilities for the crew at Colombo as well as at Kataragama Suggest an optimal assignment of the crew.