3 DIMENSIONAL GEOMETRY

Model Questions With Solutions

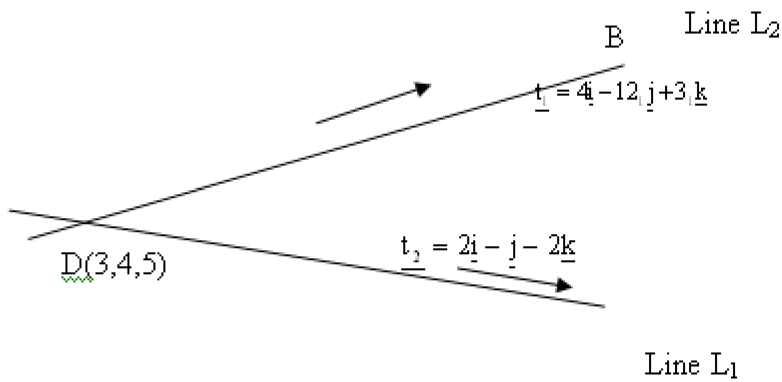
Q(1). Find the equations of the line joining (1,2,3) and (-3,4,3) and show that it is perpendicular to Z axis

Solution: Equation of st : line $\frac{x-1}{4}=\frac{y-2}{-2}=\frac{z-3}{0}=t$, a vector parallel to the line is $\underline{n}=4\underline{i}-2\underline{j}$. Therefore $\underline{n}\cdot\underline{k}=0$, hence line is normal to z axis.

Q(2). Find the equation of the line bisecting the angle between the lines

$$\frac{x-3}{2}=\frac{y+4}{-1}=\frac{z-5}{-2}, \quad \frac{x-3}{4}=\frac{y+4}{-12}=\frac{z-5}{3}$$

Solution: Let $\underline{t}_1$ and $\underline{t}_2$ be parallel vectors to the given two lines L_1 and L_2 respectively.



$\underline{t}_1=4\underline{i}-12_1\underline{j}+3_1\underline{k}$ and $\underline{t}_2=2\underline{i}-\underline{j}-2\underline{k}$

Therefore unit vector parallel to the same lines are $\frac{2\underline{i}-\underline{j}-2\underline{k}}{13}$ and $\frac{4\underline{i}-12_1\underline{j}+3_1\underline{k}}{3}$

Hence vector along the lines of bisectors

$$\frac{2\underline{i}-\underline{j}-2\underline{k}}{13} \pm \frac{4\underline{i}-12\underline{j}+3\underline{k}}{3} \quad \text{i.e,} \quad \frac{38\underline{i}-49\underline{j}-17\underline{k}}{39} \quad \text{and} \quad \frac{14\underline{i}+23\underline{j}-35\underline{k}}{39}$$

$\therefore$ Equations of bisectors through lines are $\frac{x-3}{38}=\frac{y+4}{-49}=\frac{z-5}{-17}=t$ and

$$\frac{x-3}{14}=\frac{y+4}{23}=\frac{z-5}{-35}=k.$$

Q(3) Show that the lines $\frac{x+3}{3}=\frac{y+3}{5}=\frac{z+5}{7}$ and $\frac{x-2}{1}=\frac{y-4}{5}=\frac{z-6}{7}$ intersect and find the co-ordinates of the point of intersection

Solution

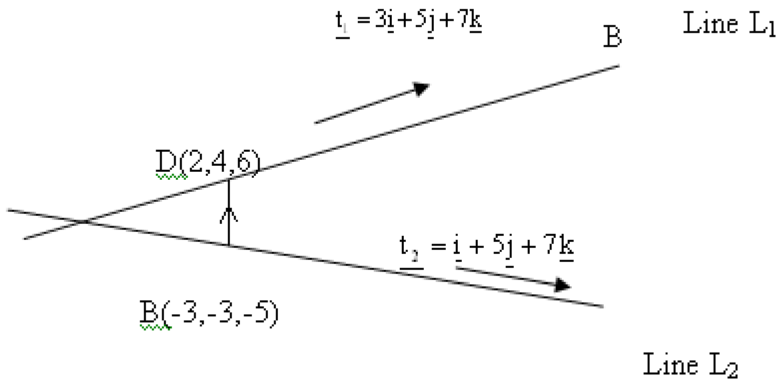
Suppose two lines L_1 and L_2 intersect, then lines L_1 and L_2 are coplanar lines.

Let $\underline{t}_1$ $\underline{t}_1$ and $\underline{t}_2$ be parallel vectors to the given two lines L_1 and L_2 respectively.

Therefore $\underline{t}_1$, $\underline{t}_2$ and

$\overrightarrow{BD}$ are also coplanar vectors. Where $\overrightarrow{BD} = 5\underline{i}+7\underline{j}+11\underline{k}$

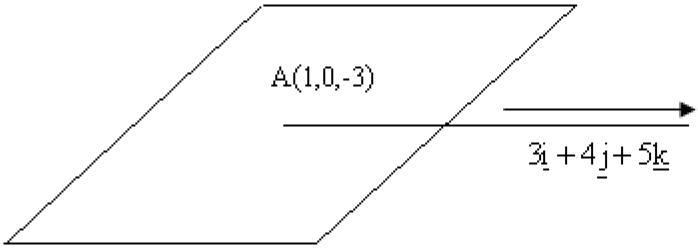
$\Rightarrow (\underline{t}_1 \times \underline{t}_2)\cdot\overrightarrow{BD} = 0$



$$\begin{vmatrix} 3 & 5 & 7 \\ 1 & 5 & 7 \\ 5 & 7 & 11 \end{vmatrix} \neq 0$$ $\therefore$ lines L_1 and L_2 are not coplanar and point of intersection does not exists.

Q(4). Find the equations of the perpendicular from (1, 0, -3) to the line

$$\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$$



Solution: Let a line through (1,0,3) which is perpendicular to he vector $3\underline{i}+4\underline{j}+5\underline{k}$

can be written as in the form $\frac{x-1}{l}=\frac{y-0}{m}=\frac{z+3}{n}=k$ with $l\underline{i}+m\underline{j}+n\underline{k}$ normal to

$3\underline{i}+4\underline{j}+5\underline{k}$

$\therefore 3l+4m+5n=0$ One can choose arbitrarily l, m, n values so that $3l+4m+5n=0$

i.e, l =4, m =-3 , n=0 or l = 2, m =6, n = -6

$$\frac{x-1}{4}=\frac{y-0}{-3}=\frac{z+3}{0}=t \quad \text{or} \quad \frac{x-1}{2}=\frac{y-0}{6}=\frac{z+3}{-6}=k$$

Q(5). Find the co-ordinates of the foot of the perpendicular from P(1, 2, 3) to the line

$\frac{x-2}{1}=\frac{y-1}{2}=\frac{z}{3}$ Find the length of the perpendicular and its equations

Solution: Coordinates of the foot of the perpendicular $\left(\frac{19}{7},\frac{17}{7},\frac{15}{7}\right)$

Q(6) Find the points on the lines $\frac{x-6}{3}=\frac{y-7}{-1}=\frac{z-4}{1}$ and $\frac{x}{-3}=\frac{y+9}{2}=\frac{z-2}{4}$ which are nearest to each other. Hence find the shortest distance between the lines and its equation..

Solution: coordinates of the nearest points are $(3, 8, 3)$, $(-3, -7, 6)$, shortest distance = $\sqrt{270}$, Equation of the shortest distance is $\frac{x-3}{6}=\frac{y-8}{15}=\frac{z-3}{3}=k$.

Q(7). Find the shortest distance between the lines $\frac{x-3}{1}=\frac{y-5}{-2}=\frac{z-7}{1}$ and $\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$.

Solution shortest distance between two lines = $2\sqrt{29}$,

Q(8). Prove that the shortest distance between the diagonal of the rectangular parallelepiped and the edges not meeting it are $\frac{bc}{\sqrt{b^2+c^2}}, \frac{ca}{\sqrt{a^2+c^2}}, \frac{ab}{\sqrt{b^2+a^2}}$ where a,b, and c are length of the edges.

Q(9). Find a vector normal (perpendicular) to the plane of P(1,-1, 0), Q(2, 1,-1) and R(-1, 1, 2).

Solution A vector normal to the plane is $6\underline{i}+6.$

Q(10). Find the vector equation of the line through the point with position vector $2\underline{i}-\underline{j}-3\underline{k}$ which is parallel to the vector $\underline{i}$ + $\underline{j}$ + $\underline{k}.$ Determine the points corresponding to $\lambda = 3, \; 0, \; 2$ in the resulting equation

Solution: A vector equation of the line is $\underline{r} = 2\underline{i}-\underline{j}-3\underline{k}+\lambda(\underline{i}+\underline{j}+\underline{k})$ and the corresponding points are $\lambda = 0, \; \underline{r} = 2\underline{i}-\underline{j}-3\underline{k}$

$\lambda = 3, \; \underline{r} = 5\underline{i}+2\underline{j})$

$\lambda = 2, \; \underline{r} = 4\underline{i}+\underline{j}-\underline{k})$

Q(1).

Solution:

Angle between the lines is (i) $\theta = \cos^{-1}\left(-\frac{1}{6}\right)$ (ii) $\theta = \frac{\pi}{2}$, (iii) $\theta = \cos^{-1}\left(\frac{4}{21}\right)$

Q(2). Find the shortest distance between the lines $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$ and $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$.

$\underline{a} = 4\underline{i} + 6\underline{i} + 2\underline{k}$, $\underline{b} = 3\underline{i} + 10\underline{i} + 5\underline{k}$, and $\underline{c} = -4\underline{i} + 5\underline{i} + 5\underline{k}$

$(\underline{a} \times \underline{b}) \cdot \underline{c} = \begin{vmatrix} 4 & 6 & 2 \\ 3 & 10 & 5 \\ -4 & 5 & 5 \end{vmatrix} = 0$ Therefore, $(\underline{a} \times \underline{b}) \cdot \underline{c} = 0$ implies that these four points are coplanar.

Q(3) . Equation of the plane is $9x + 8y - 3z = 38$.

Q(4). Equations of the plane are (i) $x + 5y - 6z = -19$. (ii) $5x - 6y + 7z = 20$, (iii) $x + y + z = 9$.

Q(5). Equation of the plane is $71x + 19y - 16z = 57$.

Q(6). (i) Equation of st : line $\frac{x-1}{4} = \frac{y-2}{-2} = \frac{z-3}{0} = t$, a vector parallel to the line is $\underline{n} = 4\underline{i} - 2\underline{j}$. Therefore $\underline{n} \cdot \underline{k} = 0$, hence line is normal to z axis.

(ii) Distance between two points is 13.

Q(7). Equation of st : line $\frac{x+1}{1} = \frac{y-3}{2} = \frac{z-2}{2} = t$,coordinates of the foot is $\left(\frac{-5}{3}, \frac{5}{3}, \frac{2}{3}\right)$ Image point is $\left(\frac{34}{11}, \frac{65}{11}, \frac{-8}{11}\right)$.

Q(9). Equations of bisectors through lines are $\frac{x-3}{38} = \frac{y+4}{-49} = \frac{z-5}{-17}$ and $\frac{x-3}{14} = \frac{y+4}{23} = \frac{z-5}{-35}$

Q(10). Symmetrical equation of line of projection is $\frac{x-1/3}{10}=\frac{y}{78}=\frac{z+1/3}{2}$

Q(11) (i) Equation of plane is $-3x+y+4z=11.$ (ii) Angle between lines is

$$\theta=\cos^{-1}\left(\frac{7}{\sqrt{14\times 101}}\right)$$

Q(14). Equation of st : line $\frac{x}{4}=\frac{y}{21}=\frac{z}{-7}=t$.