UNIVERSITY OF MORATUWA

MSC/POSTGRADUATE DIPLOMA IN OPERATIONAL RESEARCH

POR(502) OPERATIONAL RESEARCH TECHNIQUE 1

THREE HOURS **AUGUST 2007**

Answer **FIVE** questions and **NO MORE**.

Question 1

Solve the following LPP

(a) $\text{Max}Z = 3x_1 + 2x_2 + 5x_3$

Subject to

$x_1 + 2x_2 + x_3 \leq 430$

$3x_1 + 2x_3 \leq 460$

$x_1 + 4x_2 \leq 420$

$x_1,\ x_2,\ x_3 \geq 0$

(b) 8.An aero plane can carry a maximum of 200 passengers. A profit of Rs.400/= is made on each first class ticket and a profit of Rs.300 /= on each economy class ticket. The airplane reserves at least 20 seats for first class. However, at least 4 times as many passengers prefer to travel by economy class than by the first class.

(i) Formulate the associated maximization problem as a linear programming problem.
(ii) Using the graphical method, determine how many of each type of tickets must be sold to maximize the profit.

Question 2

(b) A company has a daily budget of 320 hours of labour and 350 units of raw material to manufacture two products. If necessary, the company can employ up to 10 hours daily of overtime labour hours at the additional cost of $ 2 an hour. It takes 1 labour hour and 3 units of raw material to produce one unit of product 1, and 2 labour hours and 1 unit of raw material to

produce 1 unit of product 2. The profit per unit of product 1 is $ 10, and that of product 2 is $ 12, Let x_1 and x_2 define the daily number of unit produced of products 1 and 2 ,, and x_3 the daily hours of overtime used. The LP model and its associated optimal simplex tableau are then given as

Maximize $z = 10x_1 + 12x_2 - 2x_3$

Subject to

$x_1 + 2x_2 - x_3 \leq 320$ (Labour hours)

$3x_1 + x_2 \leq 350$ (Raw material)

$x_3 \leq 10$ (Overtime)

$x_1, 2x_2, x_3 \geq 0$

Basic	x_1	x_2	x_3	S_1	S_2	S_3	Value
z	0	0	0	26/5	5/8	10/16	2256
x_2	0	1	0	3/5	-1/5	3/5	128
x_1	1	0	0	-1/5	2/5	1/5	74
x_3	0	0	1	0	0	1	10

(i) Determine the optimal solution of the problem.

(ii) Determine the dual prices.

(iii) The company currently pays an additional $ 2 per overtime hour. What is the most the company should be willing to pay?

(iv) If the company can acquire an additional 100 units of raw material daily at $ 1.50 a unit, would you advice the company to do so? What is the cost of raw material is $ 2 a unit.

(v) Suppose that the company can acquire at most 200 additional units of raw material a day, determine the associated optimal solution.

(b) A company possesses two manufacturing plants, each of which can produce three products X, Y, Z from common raw material. However, the proportion in which the products are produced are different in each plant and so are the plants operating cost pr hour. Data on production per hour and cost are given below together current orders in hand for each product.

	Product			Operating cost
	X	Y	Z	per hour (Rs)
Plant 1	2	4	3	9
Plant 2	4	3	2	10
Order on hand	50	24	60	

Use appropriate method to find the number of production hours needed to fulfill the orders on hand on a minimum cost.

Question 3

(a) A college is having an undergraduate programme for which the effective semester time available is very less and the degree course requires field work. Hence, the savings in the total number of class hours handled can be utilized for such field work. Based on past experience, the college has established the number of hours required by each faculty to teach each subject. The course in its present semester has 4 subjects and the college has considered 6 existing faculty members to teach these courses. The objective is to assign the best 4 teachers, out of these 6 faculty to teach 4 different subjects such that the total number of class hours required is minimized. The data for this problem is summarized below. Solve and optimize the assignment problem.

		Subject			
		1	2	3	4
Faculty	1	25	44	33	35
	2	33	40	40	43
	3	40	35	33	30
	4	44	45	28	35
	5	45	35	38	40
	6	40	49	40	46

(b) A company has received a contract to supply gravel for three new construction projects located in towns *A*, *B* and C. Construction engineers have estimated the required amounts of gravel which will be needed at these construction projects as shown below:

Project location	Weekly requirement (truck loads)
A	72
B	102
C	41

The company has three gravel plants *X*, *Y* and Z located in three different towns. The gravel required by the construction projects can be supplied by these three plants. The amount of gravel which can be supplied by each plant is as follows:

Plant	Amount available per/week (truck loads)
X	76
Y	62
Z	77

The company has computed the delivery cost from each plant to each project site. These costs (in rupees) are shown in the following table:

		Cost per truck load		
		A	B	C
Plant	X	4	8	8
	Y	16	24	16
	Z	8	16	35

(i) Schedule the shipment from each plant to each project in such a manner so as to minimize the total transportation cost within the constraints imposed by plant capacities and project requirements.

(ii) Find the minimum cost.

(iii) Is the solution unique? If it is not, find alternative schedule with the same minimum cost.

Question 4

Two players A and B play a zero sum game in which A has strategies A_1, A_2, A_3 and B has strategies B_1,B_2, B_3 . The payoff to A when A plays strategy A_i and B plays strategy B_j is given by the entry in row I and column j of the following matrix D.

		Player B			
		1	2	3	4
Player A	1	3	0	3	5
	2	1	5	2	0
	3	5	6	6	1

(i) Show that this game is not stable.

(ii) Explain carefully how the matrix D can be reduced by dominance to a 2x2 matrix.

(iii) Develop a Linear programming model with respect to Player A to find optimal strategies.

(iv) Develop a Linear programming model with respect to Player B to find optimal strategies.

(v)The final tableau and the corresponding feasible dictionary for the solution of the linear programming formulation of B's problem is given below, S_1 and S_2 are slack are the slack variables in the first and second constraints respectively.

Basic	x_1	x_2	x_3	S_1	S_2	Value
z	1/3	0	0	1/6	1/6	1/3
x_3	3/5	0	1	1/5	0	1/5
x_2	11/15	1	0	-1/30	1/6	2/15

Find from this tableau, the optimal mixed strategy for each player in the reduced game.

Hence find the optimal mixed strategy fpor each player in the original game and state the value of the game.

Question 5

Consider the following data of project.

Activity	Normal		Crash	
	Time(weeks)	Cost (Rs)	Time(weeks)	Cost (Rs)
1-2	7	600	4	840
1-3	11	200	9	First week Rs 70 Second week Rs 80
2-3	10	800	8	1000
2-4	6	500	4	760
2-5	16	100	9	380
3-4	6	200	4	360
3-5	9	500	4	960
4-5	8	300	5	500

The direct cost per week is Rs 300.

(a) Construct the project net work.

(b) Find the critical path.

(c) Find the normal project cost and normal project duration.

(d) Find the optimal crashed project completion time.