UNIVERSITY OF MORATUWA

MSC/POSTGRADUATE DIPLOMA IN OPERATIONAL RESEARCH

MA 5002 OPERATIONAL RESEARCH TECHNIQUES 1

THREE HOURS **AUGUST 2009**

Answer **FIVE** questions and **NO MORE**.

Question 1

The Hotel Corporation has just purchased a small hotel for conversion to mini apartments. The building, in a popular area of Washington, DC, near the U.S. State Department, will be highly marketable, and each mini apartment sale is expected to yield a good profit: The conversion process, however, includes several options. Basically, four types of mini apartments can be designed out of the former hotel rooms. They are deluxe one-bedroom apartments, regular one-bedroom apartments, deluxe studios, and efficiency apartments. Each will yield a different profit, but each *type* also requires a different level of investment in carpeting, painting, appliances, and carpentry work. Bank loans dictate a limited budget that may be allocated to each of these needs. Profit and cost data, and cost of conversion requirements, for each apartment are shown in the Table 1.

Table 1 **Type of Apartment**

	Deluxe	regular	deluxe	Efficiency	total budget
Renovation	one-bedroom	one-bed room	studio		
Requirement	($)	($)	($)	($)	($)
New carpeting	1,100	1,000	600	500	35,000
Painting	700	600	400	300	28,000
New appliances	2,000	1,600	1,200	900	45,000
Carpentry work	1,000	400	900	200	19,000
Profit per unit	8,000	6,000	5,000	3,500	

Thus, we see that the cost of carpeting a deluxe one bedroom unit will be $1,100, the cost of carpeting a regular one-bedroom unit is $1,000, and so on .A total of $35,000 is budgeted for all new carpeting in the building.

Zoning regulations dictate that the building contain no more than 50 condominiums when the conversion is completed-and no less than 25 units. The development company also decides that to have a good blend of owners, at least 40% but no more than 70% of the units should be one-bedroom apartments. Not all money budgeted in each category need be spent, although profit is not affected by cost savings. But since the money represents a bank loan, under no circumstances may it be exceeded or even shifted from one area, such as carpeting, to another, such as painting.

(a) Formulate Hotel Corporation's decision as a linear program to maximize profits.

(b) Convert your objective function and constraints to a form containing the appropriate slack, surplus, and artificial variables.

Question 2

(a) An air conditioning manufacturer produces room air conditioners at plants in Houston, Phoenix, and Memphis. These are sent to regional distributors in Dallas, Atlanta, and Denver. The shipping cost:; vary, and the company would like to find the least-cost way to meet the demands at each of the distribution centers. Dallas needs to receive 800 air conditioners per month, Atlanta needs 600, and Denver needs 200. Houston has 850 air conditioners available each month, Phoenix has 650, and Memphis has 300. The shipping cost per unit from Houston to Dallas is $8, to Atlanta is $12, and to Denver is $10. The cost per unit from Phoenix to Dallas is $10, to Atlanta is $14, and to Denver is $9. The cost per unit from Memphis to Dallas is $11, to Atlanta is $8, and to Denver is $12. How many units should be shipped from each plant to each regional distribution center? What is the total cost for this?

(b) A Company has taken the third floor of a multi-storeyed building for rent with a view to locate one of their zonal offices. There are five main rooms in this floor to be assigned to five managers. Each room has its own advantages and disadvantages. Some have windows, some are closer to the washrooms or to the canteen or secretarial pool. The rooms are all of different sizes and shapes. Each of the five managers were asked to rank their room preferences amongst the rooms 301, 302, 303, 304 and 305.

Their preferences were recorded in a Table as indicated below.

Table

Manager

M_l,	M_2	M_3	M_4	M_5
302	302	303	302	301
303	304	301	305	302
304	305	304	304	304
	301	305	303	
		302		

Most of the managers did not list all the five rooms since they were not satisfied with some of these rooms and they left off these from the list. Assuming that their preferences can be quantified by numbers, find out as to which manager should be assigned to which room so that their total preference ranking is minimum.

Question 3

The following mathematical formulation describes a problem of allocating three resources to the annual production of four commodities by a manufacturing firm. The amounts of four products to be produced are represented by x_i, x_2, x_3 and x_4 respectively. The objective function reflects the profit contribution in rupees of these products.

Maximise $z = 2x_1 - x_2 + 3x_3 - 2x_4$

subject to

$x_1 + 3x_2 - x_3 + 2x_4 \le 7$ (resource A)

$-x_1 + 2x_2 + 4x_3 \le 12$ (resource B)

$-x_1 - 4x_2 + 3x_3 + 8x_4 \le 10$ (resource C).

If you add s_1, s_2 and s_3 as slack variables, you have at the final iteration of the simplex method,

B.V.	x_1	x_2	x_3	x_4	s_1	s_2	s_3	rhs
Z	0	46/3	0	34/3	11/3	5/3	0	137/3
x_1	1	14/3	0	8/3	4/3	1/3	0	40/3
x_2	0	5/3	1	2/3	1/3	1/3	0	19/3
s_3	0	-13/3	0	26/3	1/3	-2/3	1	13/3

where Z represents the profit of the program.

(i) State optimal values for each x_j and the objective function. Is the optimal solution unique ?

(ii) For each of the variables x_1 and x_3 give an interval for its objective function coefficients such that the basic solution in part (i) remains optimal.

(iii) If the availability of resource A is increased from 7 units to 8 units, what would happen to the value of Z|?

(iv) Write the dual problem. Indicate optimal values for the dual variables and calculate the associated value of the dual objective function.

(v) Write an economic interpretation of each dual variable.

(vi) Suppose the Research and Development department proposes a new product x_5 whose production coefficients in the constraints for resources A, B and C are 5, -3 and I respectively and the objective function coefficient is 2, can the solution in part (i) be improved ? If so, show how ? If not, indicate what happens if x_5 is introduced into the basis.

(vii) How much could the right-hand side in constraint number 2 be decreased before profit would be affected?

Question 4

A manufacturer produces three products *A, B* and C. Each product requires processing on two machines I and Il. The time required to produce one unit of each product on a machine is Time to produce one unit (hrs.) Product

	Machine I	Machine II
A	0.5	0.6
B	0.7	0.8
C	0.9	1.05

There are 850 hours available on each machine. The operating cost is Rs. 5/hr. for machine I and Rs. 4/hr. for machine II. The market requirements are at least 90 units of *A,* at least 80 units of B and at least 60 units of C. The manufacturer wishes to meet the requirements at minimum cost. Solve the problem by the simplex method.

Question 5

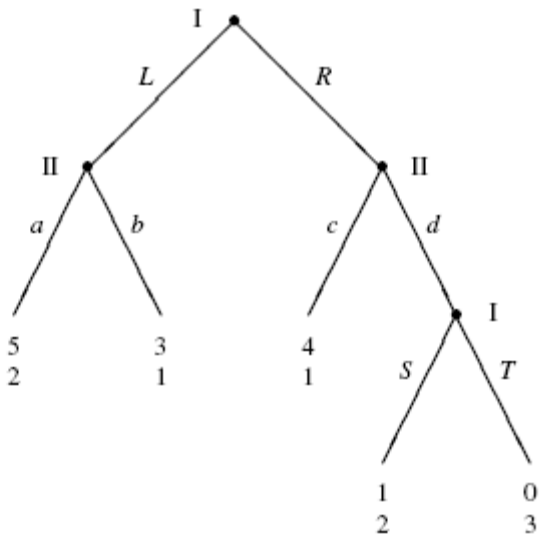

Figure 1

Consider the game tree in Figure 1. At a leaf, the top payoff is for player I, the bottom payoff for player II.

(a) What is the number of strategies of player I and of player II?

(b) How many reduced strategies does each of the players have?

(c) Give the reduced strategic form of the game.

(d) What are the Nash equilibria of the game in reduced strategies? What are the subgame perfect equilibria of the game?

(e)Identify every pair of reduced strategies where one strategy weakly or strictly dominates the other, and indicate if the dominance is weak or strict.

Question 6

Solve the following L.P.P. by the simplex method:

$$\begin{aligned} &\text{Minimize} && Z = 3x_1 + 4x_2 + 5x_{3,} \\ &\text{Subject to} && 2x_1 + x_2 + 2x_3 \geq 30, \\ & && 2x_1 + 3x_2 + x_3 \leq 36, \\ & && 2x_1 + x_2 + x_3 \geq 35, \\ & && x_{1,} x_{2,} x_3 \geq 0. \end{aligned}$$

(a) Identify the basic and non- basic variables in the solution.

(b) What are the base row coefficients of non-basic variable?

(c) How are they useful for sensitivity analysis increases by 5 each of the resources.

Question 7

(a) The table below provides cost and estimates for a seven activity project.

Activity	Time estimate (weeks)		Direct cost estimate (Rs. 1000)	
	Normal	Crash	Normal	Crash
A (1-2)	2	1	10	15
B (1-3)	8	5	15	21
C (2-4)	4	3	20	24
D (3-4)	1	1	7	7
E (3-5)	2	1	8	15
F (4-6)	5	3	10	16
G (5-6)	6	2	12	36

(i) Draw the project network corresponding to normal time.

(ii) Determine the critical path and the normal duration and cost of the project.

(iii) Crash the activities so that the project completion time reduces to 9 weeks.

(b)A network with the following activity durations and manpower requirement is given. Analyze the project from point of view of resources to bring out the necessary steps involved in the analysis and smoothing of resources so that minimise number of men required to each actvity.

Activity	1-2	2-3	2-4	3-5	4-6	4-7	5-8	6-8	7-9	8-10	9-10
Duration (weeks)	2	3	4	2	4	3	6	6	5	4	4
No. of men required	4	3	3	5	3	4	3	6	2	2	9

Question 8

Two players A and B play a zero sum game in which A has strategies A_1, A_2, A_3 and B has strategies B_1,B_2, B_3 . The payoff to A when A plays strategy A_i and B plays strategy B_j is given by the entry in row I and column j of the following matrix D.

		Player B			
		1	2	3	4
Player A	1	3	0	3	5
	2	1	5	2	0
	3	5	6	6	1

(i) Show that this game is not stable.

(ii) Explain carefully how the matrix D can be reduced by dominance to a 2x2 matrix.

(iii) Develop a Linear programming model with respect to Player A to find optimal strategies.

(iv) Develop a Linear programming model with respect to Player B to find optimal strategies.

(v)The final tableau and the corresponding feasible dictionary for the solution of the linear programming formulation of B's problem is given below, S_1 and S_2 are slack are the slack variables in the first and second constraints respectively.

Basic	x_1	x_2	x_3	S_1	S_2	Value
z	1/3	0	0	1/6	1/6	1/3
x_3	3/5	0	1	1/5	0	1/5
x_2	11/15	1	0	-1/30	1/6	2/15

Find from this tableau, the optimal mixed strategy for each player in the reduced game.

Hence find the optimal mixed strategy for each player in the original game and state the value of the game.