



UNIVERSITY OF MORATUWA

MSC/POSTGRADUATE DIPLOMA IN OPERATIONAL RESEARCH

MA 5006 NUMERICAL METHODS

THREE HOURS

SEPTEMBER 2009

Answer **FIVE** questions and **NO MORE**.

Question 1

(a) Write down the five rules for good programming practice?

Why would you use an M-file as opposed to entering commands into the command window?

(b) For each of the following requirements, write a 'snippet' of MATLAB code which can achieve the requirement:

(i) Obtain the user's student ID number interactively pause the execution of the code for 20 seconds before displaying the student ID number back to the screen.

(ii) Obtain the value of an angle in degrees from the user and display the corresponding value in radians back to the screen.

Question 2

The series representation of a function for the study of the movement of hanging chains is given by

$$B(x) = \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}x\right)^k}{(k!)^2}.$$

(a) Obtain approximations for $B(2)$ using first two and three terms of the series. You may express your results as fractions.

(b) Estimate the absolute error in your approximations when taking three terms, indicating clearly the method employed.

(c) Complete the following function-file **Bser** to approximate the function at x using a given number of terms.

```
function [Bapprox,Ea]=Bser(x,n)
% This function evaluates the series
% B(x) to n terms
% Input :x-argument of B(x) function
%       :n-required number of terms
% Output:Bapprox-approximate value of B(x)
%       :Ea-estimated magnitude of absolute
%       error in approximation
%Initialization
Bapprox=1;      %      k=0 term
```

(d) Write down the **MATLAB** commands necessary to use this function to evaluate $B(2)$ using three terms of the series, and to obtain an estimate of the associated absolute error.

Question 3

The Newton Raphson method is one of the open methods for finding the roots of equations.

Complete the following **MATLAB** function file that implements the Newton Raphson method.

Your function file should agree with the requirement given below.

- It supplies default values if `maxit` and `es` are not supplied.

```
function root = newtraph(func,dfunc,xguess,es,maxit)
% root = newtraph(func,dfunc,xguess,es,maxit):
%       uses Newton-Raphson method to find the root of a
%       function
% input:
%   func   = name of the function
%   dfunc  = name of the derivative of the function
%   xguess = initial guess
%   es     = (optional) stopping criterion
%   maxit  = (optional) maximum allowable iterations
% output:
%   root   = real root
```

How do you use the above function file to determine a root of $f(x) = -2x^6 - 1.5x^4 + 10x + 2$ in the interval $[1, 2]$.

Question 4

The amount of a uniformly distributed radioactive contaminant contained in a closed reactor is measured by its concentration C (Becquerel/liter or Bq/L). The contaminant decreases at a decay rate proportional to its concentration; that is,

$$\frac{dC}{dt} = -kC \quad \text{where } k \text{ is a constant with units of days}^{-1}.$$

A user may enter in both k and the initial concentration C_0 . A MATLAB program is required which will provide the concentration at some later time. In order to approximate the derivative term, 'Eulers' method is proposed, where Eulers' method states that:

$$\frac{dC}{dt}(t_i) \approx \frac{C_{i+1} - C_i}{t_{i+1} - t_i}$$

The concentration at a later time, C_{i+1} may be calculated using the values C_i and the time-step $(t_{i+1} - t_i)$.

- Write down a MATLAB script file which can determine the concentration, given that the user enters the initial concentration, and the time when the new concentration is required.
- How do you use your script file to solve the equation from $t=0$ to $1 d$ with $k=0.2$. Employ a step size of $\Delta t = 1$. The concentration at $t=0$ is $10 Bq/L$.

Question 5

- Derive normal equations for the least squares polynomial of degree one.
- This question involves the relationship between the surface areas of human beings and their weight. For several individuals of the same height, in the following table x represents the weight [kg] and y represents the surface area [m^2]. Fit a function of the form $f(x) = a_1x + a_0$ to the data in the table using linear regression.

x	y
50	2.1
65	2.2
80	2.3

Would you expect this formula to be valid for a wide range of x ?

(c) A company buys a large number of hard drives for its computers. The company monitors the number(N) of drives still working as time passes. Carrying out a linear regression, they find that

$$\ln N = 10 - \frac{t}{2}, \text{ where } t \text{ is elapsed time in years.}$$

What is the explicit expression for N in terms of t ?

Question 6

In order to use Gaussian Elimination to solve a set of linear simultaneous Equations for three unknowns x_1, x_2, x_3 , the augmented matrix A_0 shown below was constructed. The Gaussian Elimination procedure was then followed as shown, where the augmented matrix was changed over six steps, to finally obtain the matrix A_6 , where the square sub matrix is diagonal. No pivoting was performed.

(a) Calculate the three unknowns x_1, x_2 and x_3 .

(b) Six of the numbers have been omitted from the calculations, in matrices A_4 and A_5 . Fill in the spaces labeled u_1, u_2, u_3, u_4, u_5 and u_6 .

$$A_0 = \left[\begin{array}{ccc|c} -3 & 4 & 7 & 7 \\ 5 & 7 & -1 & 40 \\ 2 & -5 & 1 & -6 \end{array} \right]$$

$$A_1 = \left[\begin{array}{ccc|c} 5 & -6.67 & -11.67 & -11.67 \\ 5 & 7 & -1 & 40 \\ 2 & -5 & 1 & -6 \end{array} \right]$$

$$A_2 = \left[\begin{array}{ccc|c} 5 & -6.67 & -11.67 & -11.67 \\ 0 & 13.67 & 10.67 & 51.67 \\ 2 & -5 & 1 & -6 \end{array} \right]$$

$$A_3 = \left[\begin{array}{ccc|c} 2 & -2.67 & -4.67 & -4.67 \\ 0 & 13.67 & 10.67 & 51.67 \\ 2 & -5 & 1 & -6 \end{array} \right]$$

$$A_4 = \left[\begin{array}{ccc|c} 2 & -2.67 & -4.67 & u_1 \\ 0 & 13.67 & 10.67 & u_2 \\ 0 & -2.33 & 5.67 & u_3 \end{array} \right]$$

$$A_5 = \left[\begin{array}{ccc|c} 2 & -2.67 & -4.67 & -4.67 \\ 0 & u_4 & u_5 & u_6 \\ 0 & -2.33 & 5.67 & -1.33 \end{array} \right]$$

$$A_6 = \left[\begin{array}{ccc|c} 2 & -2.67 & -4.67 & -4.67 \\ 0 & -2.33 & -1.82 & -8.81 \\ 0 & 0 & 7.49 & 7.48 \end{array} \right]$$

(ii) Use Gauss-Seidal method by taking $x_2 = 0, x_3 = 0$ as the initial approximations (carryout three iterations) to solve the following system of equations.

$$27x_1 + 6x_2 - x_3 = 85$$

$$6x_1 + 15x_2 + 2x_3 = 72$$

$$x_1 + x_2 + 54x_3 = 110$$

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