UNIVERSITY OF MORATUWA

MSC/POSTGRADUATE DIPLOMA IN OPERATIONAL RESEARCH 2006/2007

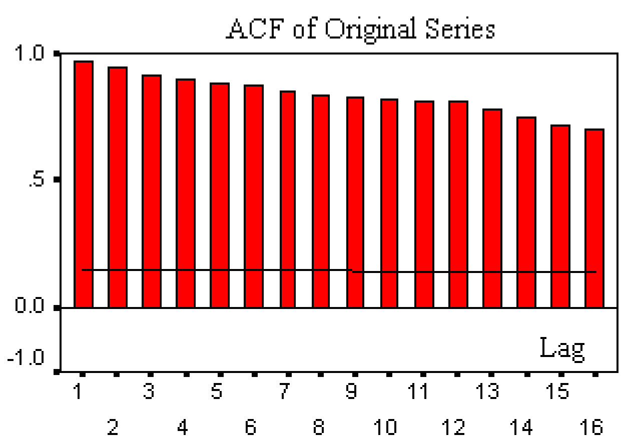
POR(509) TIME SERIES ANALYSIS & ECONOMETRICS

THREE HOURS **October 2008**

Answer **FIVE** questions and **NO MORE**.

Question 1

Miller and Wichern reported 178 monthly values $x_1, x_2, x_3, x_4, \ldots\ldots x_{178}$ of the number of people in Wisconsin employed in 'Trade' from 1961 to 1975 (all observations are given in units of 1000 employees). In Figures 1 is presented the output of the Sample Autocorrelation function(ACF) of the original values. Fig: 2 is presented ACF of first difference series $y_t = x_t - x_{t-1}$,while Fig: 3 present seasonal difference of series x_t, where $z_t = x_t - x_{t-12}$



Fig; 1 ACF of Original Series

(a)Consider Fig: 1 which shows the ACF of the original series. Describe the behavior of the original series. Should the values produced by the x_t be considered stationary? Explain your answer theoretical concepts.

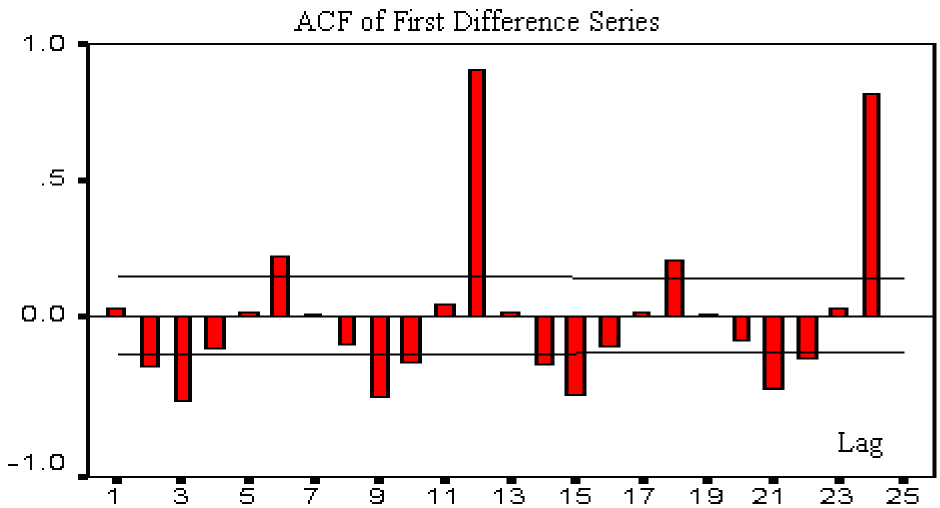


Fig:2 ACF of First Difference Series $y_t = x_t - x_{t-1}$

(b) Consider Fig: 2 which shows the ACF of the values produced by the transformation $y_t = x_t - x_{t-1}$. Describe the behavior of the first difference series. Should the values produced by the transformation $y_t = x_t - x_{t-1}$. be considered stationary? Explain your answer as in part(a).

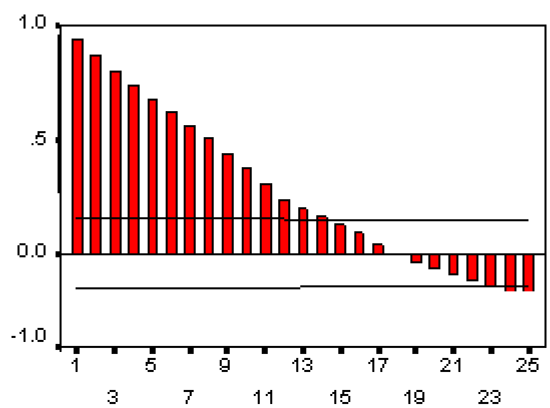


Fig: 3 ACF of Seasonally Difference Series $z_t = x_t - x_{t-12}$

(c) Consider Figure 3 which shows the ACF of the values produced by the transformation $z_t = x_t - x_{t-12}$. Describe the behavior of the seasonal difference series. Should the values produced by the transformation $z_t = x_t - x_{t-12}$ be considered stationary? Explain your answer.

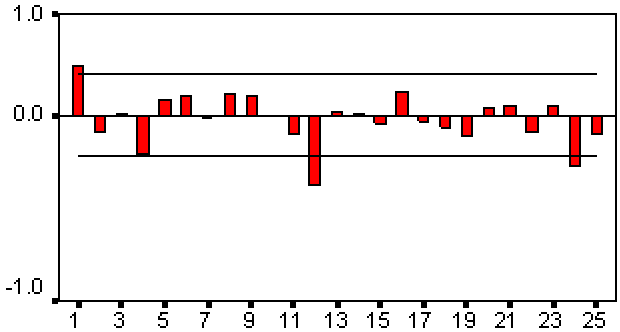
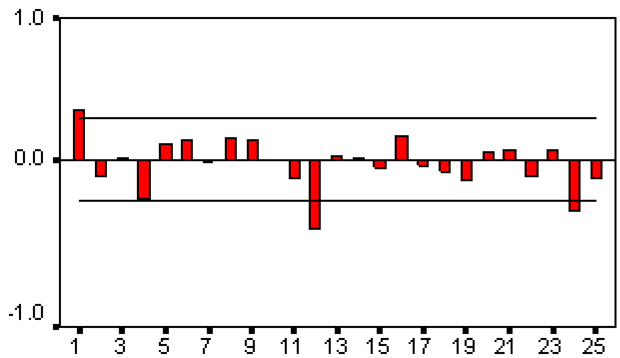


Fig: 4 ACF of Seasonally Difference of First Difference Series $w_t = x_t - x_{t-1} - x_{t-12} + x_{t-13}$

(d) Consider Fig: 4 which shows the ACF of the values produced by the transformation

$w_t = x_t - x_{t-1} - x_{t-12} + x_{t-13}$.Describe the behavior of the series w_t. Justify your answer.



(e) Describe the behavior of the partial autocorrelation function PACF. Justify your answer.

(f) Identify the above tentative model with your experience and explain briefly why you attained to this conclusion?

Question 2

(i) Define seasonality of given series. How can one determine if there is a seasonal component in the data?

(ii) Explain the difference between the additive and multiplicative models.

(iii) What does seasonally adjusted data mean? What are the advantages of reporting seasonally adjusted data?

(iv) Why and when is it necessary to compute a centered moving average?

TABLE 1

Week	Calls	Week	Calls	Week	Calls
1	50	5	45	9	35
2	35	6	35	10	20
3	25	7	20	11	15
4	40	8	30	12	40

(b) Emergency calls to Police Head Quarters 911 system, for the past 12 weeks are shown in Table 1.

Using the data in the Table 1, fit a multiplicative model to the data and find

(i) the estimates of the four seasonal factors,

(ii) the equation of trend line for deseasonalized data.

(iii) the forecast and the approximate 95 percent confidence interval for the 13th time period.

Question 3

(a)Define Autioregressive time series models, Moving average model and Mixed ARMA models with usual notations.

(b)State stationary condition and inevitability condition of each model.

(c)What is the difference between a causal model and stochastic time series model forecasting?

(d)Consider the following AR (2) models:

(i) $x_t - .6x_{t-1} - 0.3x_{t-2} = e_t$,

(ii) Check stationary condition of the model in (i)

(iii) Find the general expression for ρ_k.

(iv) Plot the ρ_{k}, fork = 0, 1, 2, 3.

Question 4

(a)Is the following a valid autocorrelation function for a real valued covariance stationary process? Why?

$$\rho_k = \begin{cases} 1, & \text{if } k = 0 \\ \alpha, & 1/2 < |\alpha| < 1, \text{ if } k = 1 \\ 0, & \text{if } k \geq 2 \end{cases}$$

(b) Prove that the following properties for the autocorrelation function of a given time series process .

(a) $\rho_0 = 1$,

(b) $|\rho_k| \leq 1$,

(c) $\rho_k = \rho_{-k}$.

(c) Show that the following function satisfies the properties stated in part (a), but is not an ACF for any stationary process:

$$\rho_k = \begin{cases} 1, & \text{if } k = 0 \\ 0.8, & k = 1 \\ 0.1, & k = 2 \\ 0. & \text{otherwise} \end{cases}$$

Question 5

(a)Explain briefly necessary steps that are used for model identification of time series.

(b)Consider a Seasonal autoregressive integrated moving average model

$\text{ARIMA}(0, 1, 1,)(0, 1, 1,)_{12}$ is defined by

$$\begin{aligned}(1-B)(1-B^{12})x_t &= (1-\theta_1 B)(1-\beta_1 B)e_e \\ &= w_t \text{(say)}\end{aligned}$$

where $\{e_t\}_{t=1}^{n}$ zero mean white noise with constant variance σ^2.

(i)Show that autocovarinace of w_t, are given by,

$$\begin{aligned}\gamma_0 &= (1+\theta_1^2)(1+\beta_1^2)\sigma_e^2 \\ \gamma_1 &= -\theta_1(1+\beta_1^2)\sigma_e^2 \\ \gamma_{11} &= -\theta_1\beta_1\sigma_e^2 = \gamma_{13} \\ \gamma_{12} &= -\beta_1(1+\theta_1^2)\sigma_e^2 \\ \gamma_j &= 0 \text{ otherwise}\end{aligned}$$

(ii) Hence compute ACF(ρ_k) for $\theta_1 = 0.4$ and $\beta_1 = -0.9$ and plot ρ_k against lag k.

Question 6

Consider the model:

$$Y_i = \beta X_i + e_i,$$

where β is the slope parameter and e_i is the error term.

(i)Derive the maximum likelihood estimators $\hat{\beta}$ and $\hat{\sigma}^2$ of the parameters of the β and σ^2 respectively.

(ii) Show that $\hat{\beta}$, is unbiased.

(iii) Given that E(MSE) $=\sigma^2$ where $MSE = \dfrac{\sum (Y_1 - \hat{Y})^2}{n-1}$ find the expected value of $\hat{\sigma}^2$. Is $\hat{\sigma}^2$ unbiased?

(iv)Derive the formula for variance of $\hat{\beta}$.

Question 7

(a) State with brief reason whether the following statements are true or false:

(i) In the presence of heteroscedasticity, OLS estimators are biased.

(ii) If heteroscedasticity is present, the conventional t and F tests are invalid.

(iii) Even though the disturbance term in the linear regression model is not normally distributed, the OLS estimators are still unbiased.

(iv) In the presence of autocorrelation, OLS estimators are BLUE.

(v) Because of the multicollinearity, the confidence intervals tend to be much wider.

(b) State the assumptions (7 assumptions) of simple linear regression.

(c) Draw the appropriate residual plots for each of the following situations:

(i) No systematic pattern in the errors (random).

(ii) Error variance is proportional to the explanatory variable.

(iii) Error variance is proportional to the square of the explanatory variable.

(iv) Positive autocorrelation.

(v) Negative autocorrelation.

(vi) Incorrect functional form.