

Consider the ODE $y'' = y' + y^2, y(0) = 4, y(1) = 1$.

- 1) Use $y_k'' = \frac{y_{k+1} - 2y_k + y_{k-1}}{h^2}$ and $y_k' = \frac{y_{k+1} - y_k}{h}$ with $h = \frac{1}{3}$ to derive a system of non-linear equations .
- 2) Use a suitable numerical method to solve the above system of non-linear equations to find y_1, y_2 .
- 3) Write the Cubic spline $C = C(x)$ for the function $y = y(x)$ using $y_0 = 4, y_3 = 1$ and keeping y_1, y_2 as constants. Use the ODE to determine $M_k = y''(x_k); k = 0, 1, 2, 3$.
- 4) Now use the properties of $C(x)$ to obtain a systems of non-linear equations and solve it to find y_1, y_2 .
- 5) Write the ODE as a system of first order ODEs and solve it by RK4 to find y_1, y_2 .

Note:

1. Two of exact same questions with a different ODE will be given for the Midmakeup1-part A.
2. The other two questions will be given as Midmakeup2 take home.
3. Midmakeup1-part B will be on PDEs, possibly containing material from the lecture on the same day, it will be MCQ.