Consider the ODE $y'' = \frac{3}{2}y^2, y(0) = 4, y(1) = 1$.

1) Use the second order method $y_k'' = \frac{y_{k+1} - 2y_k + y_{k-1}}{h^2}$ with $h = \frac{1}{3}$ to derive the system of non-linear equations $\frac{1}{6}y_k^2 = y_{k+1} - 2y_k + y_{k-1}; k = 1,2.$

2) Use a suitable numerical method to solve the above system of non-linear equations to find y_1, y_2 .

3) Write the Cubic spline $C = C(x)$ for the function $y = y(x)$ using $y_0 = 4, y_3 = 1$ and keeping y_1, y_2 as constants. Use the ODE to determine $M_k = y''(x_k); k = 0,1,2,3.$

4) Now use the properties of $C(x)$ to obtain a systems of non-linear equations and solve it to find y_1, y_2 .

5) Derive a modified Euler method $y_{k+2} = 2y_{k+1} - y_k + h^2 f(x_{k+1}, y_{k+1})$ to solve $y'' = f(x, y)$ using second order Taylor series and Trapezoidal rule for integration.

6) Use the above method to obtain a systems of non-linear equations and solve it to find y_1, y_2 .

7) Derive the standard Euler method to solve a system of first order ODEs. Use it to find y_1, y_2 .

8) Write the ODE as a system of first order ODEs and solve it by RK4 to find y_1, y_2 .

Note:

1. **Three of exact same questions with a different ODE will be given for the Mid1-part A.**
2. **The other five questions will be given as Mid2 take home.**
3. **Mid1-part B will be on PDEs, possibly containing material from the lecture on the same day, it will be MCQ.**