Question: Solve the ODE $y'(x) = x + y, y(1) = 2$

Method: Euler solving $y'(x) = f(x, y)$, $y(x_0) = y_0$
$y_{k+1} = y_k + hf(x_k, y_k), h = x_{k+1} - x_k$

Theorem:
1. $S = \{(x, y) | x \in \mathbb{R}, y \in \mathbb{R}, |x - x_0| \leq \delta, |y| < \infty\}$
2. $f: S \to \mathbb{R}$
3. f is Lipschitz continuous in the variable y $ie. |f(x, y_1) - f(x, y_2)| \leq L|y_1 - y_2|$
   Then

There is a unique solution to $y'(x) = f(x, y)$, $y(x_0) = y_0$ in $|x - x_0| \leq \delta$

Theorem:
1. f satisfies the conditions in the above theorem
2. $\|f''\|_\infty \leq M$
   Then

$|y(x_{k+1}) - y_{k+1}| \leq h\frac{M}{2L}(e^{L\delta} - 1)$

Method: RK2 solving $y'(x) = f(x, y)$, $y(x_0) = y_0$
$y_{k+1} = y_k + \frac{h}{6}[f_1 + 2f_2 + 2f_3 + f_4]$
$f_1 = f(x_k, y_k)$
$f_2 = f\left(x_k + \frac{h}{2}, y_k + \frac{h}{2}f_1\right)$
$f_3 = f\left(x_k + \frac{h}{2}, y_k + \frac{h}{2}f_2\right)$
$f_4 = f(x_k + h, y_k + hf_3)$