Question: Find the roots of $x = \frac{1}{2}\cos x$ on $\left[0, \frac{\pi}{4}\right]$.

Theorem:

1. $g: G \to \mathbb{C}$ and G is closed subset of $\mathbb{C}$
2. $g(G) \subseteq G$
3. g is Lipschitz continuous with a constant $0 \leq L < 1$

Then

1. There is a unique fixed point $z \in G$ of g
2. The sequence determined by $x_0 \in G$, $x_{k+1} = g(x_k)$ converges to z
3. $|x_k - z| \leq \frac{L^k}{1-L}|x_1 - x_0|$

Theorem:

1. $g: G \to \mathbb{C}$ and G is closed subset of $\mathbb{C}$
2. g is continuously differentiable
3. $|g'(x)| \leq L$ for all $x \in G$

Then

g is Lipschitz continuous with constant L

Note: Extend the ideas here to find the roots of $x = \frac{1}{3}\cos xy + \frac{1}{6}$ and $y = -\frac{1}{20}e^{-xy} - \frac{8}{20}$ on $[-1,1]^2$. Closed subset will be replaced by convex subset.