

Question: Solve $\begin{pmatrix} 898 & 198 & 46 \\ 198 & 46 & 12 \\ 46 & 12 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 285 \\ 76 \\ 18 \end{pmatrix}$

Algorithm: For solution of the system $Ax = b$ with $A = D + U + L$

1. **Jacobi method**

$$Ax = (D + U + L)x = b \Rightarrow Dx = -(U + L)x + b \Rightarrow x = -D^{-1}(U + L)x + D^{-1}b \Rightarrow x_{k+1} = Mx_k + N$$

2. **Gauss-Siedel Method**

$$Ax = (D + U + L)x = b \Rightarrow (D + L)x = -Ux + b \Rightarrow x = -(D + L)^{-1}Ux + (D + L)^{-1}b \Rightarrow x_{k+1} = Mx_k + N$$

Theorem: The following are equivalent(TFAE) for $A \in \mathbb{C}^{n \times n}$

1. $\lim_{k \rightarrow \infty} A^k = O$
2. $\lim_{k \rightarrow \infty} A^k x = \underline{0}$ for all x
3. $\rho(A) < 1$
4. There exists a matrix norm such that $\|A\| < 1$

Algorithm: To find the eigenvalues of A

1. **Power method** $y_k = Ax_k, \frac{\|y_{k+1}\|}{\|y_k\|} \rightarrow |\lambda_1| > |\lambda_i|$
2. **QR method** $A_k = Q_k R_k, A_{k+1} = R_k Q_k, A_k \rightarrow$ matrix with eigen values of A are on the diagonal