Question: Consider the following table

x_k	$f(x_k)$	$f'(x_k)$
1	2	0
2	3	1
4	6	2
5	7	0

1) Find a polynomial that passes through the points $(x_k, f(x_k))$.

2) Find a polynomial that passes through the points $(x_k, f(x_k))$ and agrees with $f'(x_k)$.

3) Find a piecewise polynomial that passes through the points $(x_k, f(x_k))$ such that $f'(x_k)$ exists and $f''(x_0) = f''(x_n) = 0$

4) Find a degree 2 polynomial $p_2(x)$ such that the sum of square error $\sum_{k=0}^{n}(f(x_k) - p_2(x_k))^2$ is minimized.

Theorem: Lagrange Interpolation

$$L(x) = \sum_{k=0}^{n} f(x_k) L_k(x)$$

Theorem: Hermite Interpolation

$$H(x) = \sum_{k=0}^{n} f(x_k)(1 - 2(x - x_k)L_k'(x))L_k^2(x) + \sum_{k=0}^{n} f'(x_k)(x - x_k)L_k^2(x)$$

Theorem: Cubic Spline Interpolation

$$C(x) = \left(\frac{(x_{k+1}-x)\left((x_{k+1}-x)^2-(x_{k+1}-x_k)^2\right)}{6(x_{k+1}-x_k)}\right) f''(x_k) + \left(\frac{(x-x_k)\left((x-x_k)^2-(x_{k+1}-x_k)^2\right)}{6(x_{k+1}-x_k)}\right) f''(x_{k+1})$$
$$+ \left(\frac{x_{k+1}-x}{x_{k+1}-x_k}\right) f(x_k) + \left(\frac{x-x_k}{x_{k+1}-x_k}\right) f(x_{k+1})$$