Question: Find a polynomial that passes through three given points. Use it to find $\int_0^1 e^{-x^2}\,dx$ with an accuracy of ± 0.01.

Theorem: Lagrange Interpolation
If $f:[a,b]\to\mathbb{R}, f\in C^{(n+1)}, x_k\in[a,b]: k=0,1,\cdots,n$
then $f(x)=\sum_{k=0}^{n} f(x_k)\prod_{k\neq j=0}^{n}\frac{(x-x_j)}{(x_k-x_j)}+\frac{f^{(n+1)}(c)}{(n+1)!}\prod_{k=0}^{n}(x-x_k)$,where $c\in(a,b)$.

Theorem: Simpson's rule
If $f:[a,b]\to\mathbb{R}, f\in C^4, h=\frac{b-a}{n}, x_0=a, x_n=b, x_k=a+kh: k=0,1,\cdots,n$ and n is the number of intervals
then $\int_a^b f(x)\,dx=\frac{h}{6}\left(f(x_0)+4\sum_{odd k=1}^{n-1} f(x_k)+2\sum_{even k=2}^{n-2} f(x_k)+f(x_n)\right)-\frac{nh^5}{180}f^{(4)}(c)$,where $c\in(a,b)$.