

17. Multiply the integrals $I = \int_0^{\infty} e^{-x^2} dx$ and $I = \int_0^{\infty} e^{-y^2} dy$, write as a double integral, convert to polar coordinates and find I .
18. Consider the double integral $\int_0^{\infty} \int_0^{\infty} e^{-xy} \sin x dx dy$, use Fubini's theorem to find $\int_0^{\infty} \frac{\sin x}{x} dx$.
19. Consider the complex integral $\int_C \frac{e^{iz}}{z} dz$ where C the boundary of the region between the circles of radius R and r with $R > r$ above the x axis. Let $R \rightarrow \infty$ and $r \rightarrow 0$ and use Cauchy's Residue theorem to find $\int_0^{\infty} \frac{\sin x}{x} dx$. Show that complex integrals become zero when necessary.
20. Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Also find the perimeter using Elliptic Integrals.
21. Find the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. Also find surface area using Elliptic Integrals, use a modified form of spherical polar coordinates.
22. Use Divergence theorem to prove the Archimedes's Principle.
23. Let $u = u(x, y), s = s(x, y), t = t(x, y)$ and $\det \frac{\partial(s,t)}{\partial(x,y)} \neq 0$.
We rewrite $Au_{xx} + Bu_{xy} + Cu_{yy}$ as $\bar{A}u_{ss} + \bar{B}u_{st} + \bar{C}u_{tt} + \dots$ after changing variables.
Show that $\bar{B}^2 - 4\bar{A}\bar{C} = \left(\det \frac{\partial(s,t)}{\partial(x,y)}\right)^2 (B^2 - 4AC)$.
24. Let \mathbf{f} be a vector field continuous on an open connected set A in \mathbb{R}^3 . Show that \mathbf{f} is the gradient of some potential function in A iff the line integral of \mathbf{f} is independent on any piecewise smooth path in A .
25. First question in MID
26. Second question in MID