1. Let : $[a, b] \to \mathbb{R}$. Prove $f \in \mathcal{R}[a, b] \Rightarrow \forall \varepsilon > 0\ \exists P \in \mathcal{P}[a, b];\ U(P, f) - L(P, f) < \varepsilon$

2. Let $f(x) = \begin{cases} 1 & , \quad x \text{ is rational} \\ 0 & , \quad x \text{ is irrational} \end{cases}$. Is f Reimann integrable?

3. Show that $\int_{0^+}^{\infty} e^{-t} t^{x-1} dt$ is converging for $x > 0$.

4. Is $\int_1^{\infty} \frac{\sin x}{x} dx$ converging/diverging?

5. Find the velocity and the acceleration along a curve described by $\boldsymbol{r} = \boldsymbol{r}(t)$. The unite tangential, normal and binormal unit vectors are $\boldsymbol{T}, \boldsymbol{N}, \boldsymbol{B}$ respectively. Deduce the results for a circle of radius a.

6. Find the time period T of a pendulum of mass m and length ℓ which is oscillating at an angle 2α.

7. Let : $A \subseteq \mathbb{R}^2 \to \mathbb{R}$. Show that $f \in \mathcal{C}^1 \Rightarrow f \in \mathcal{D} \Rightarrow f \in \mathcal{C}$

8. Let $f(x, y) = x^4 + y^4 - x^2 - y^2 + 1$. At the point $(1,2)$ find
   - Direction in which the function increases most rapidly
   - Directional derivative in that direction
   - Equation of the tangent plane.

9. Let $f: \mathbb{R}^2 \to \mathbb{R}$. Find sufficient conditions for a critical point with $\nabla f = \boldsymbol{0}$ to be a local maximum/minimum. Find the critical points of $f(x, y) = x^3 - 12x + y^3 - 27y + 5$ and classify them.

10. Find $\int_C \boldsymbol{f} \cdot d\boldsymbol{r}$ where $\boldsymbol{f}(x, y) = \sqrt{y}\boldsymbol{i} + (x^3 + y)\boldsymbol{j}$ where the path C is given by
    $C: x = t, y = t, 0 \le t \le 1$
    $C: x = t^2, y = t^3, 0 \le t \le 1$

11. Use integration by parts to find $\int_0^1 \int_{y^2}^1 y \cos(x^2)\, dx dy$.

12. Verify the Green's theorem for the region between the curves $y = x$ and $y = x^2$ form $x = 0$ to 1 for the vector field $\boldsymbol{f}(x, y) = y^2\boldsymbol{i} + x\boldsymbol{j}$.

13. Let C be the intersection of the surfaces $S_1: z = x^2 + y^2$ and $S_2: z = 1$. Find
    - Surface area of S_1.
    - Verify the Stokes theorem $\int_C \boldsymbol{f} \cdot d\boldsymbol{r} = \iint_S \operatorname{curl} \boldsymbol{f} \cdot d\boldsymbol{S}$ for $\boldsymbol{f}(x, y, z) = z^2\boldsymbol{i} + x^2\boldsymbol{j} + y^2\boldsymbol{k}$? for both surface S_1 and S_2.

14. Let V be the volume between the surfaces $S_1: z = x^2 + y^2$ and $S_2: z = 1$. Find
    - Volume of V.
    - Verify the Divergence theorem $\iint_S \boldsymbol{f} \cdot d\boldsymbol{S} = \iiint_V \operatorname{div} \boldsymbol{f}\, dV$ for $\boldsymbol{f}(x, y, z) = xy\boldsymbol{i} + yz\boldsymbol{j} + zx\boldsymbol{k}$?

15. Let $f(z) = u(x, y) + iv(x, y)$ and $u, v \in \mathfrak{D}$. Show that $f \in \mathfrak{D} \Leftrightarrow u_x = v_y$ and $u_y = -v_x$(CR equations)

16. Use complex integration to find $\int_{-\infty}^{\infty} \frac{1}{1+x^4} dx$.