Consider the Torus (donut shape) given by $a^2 = z^2 + \left(\sqrt{x^2+y^2} - c\right)^2$ with $0 < a < c$.

Q1. Show that $x = (c + a\cos v)\cos u, y = (c + a\cos v)\sin u, z = a\sin v$ where $0 \le u, v < 2\pi$ is a suitable parameterization. Draw a picture and identify a, c, u, v.

Q2. Find the position vector $\boldsymbol{r}(u, v)$ and also the partial derivatives of this position vector w.r.t the parameters u, v. i.e. $\boldsymbol{r}_u$ and $\boldsymbol{r}_v$.

Q3. Find the Matric $ds^2 = \boldsymbol{dr} \cdot \boldsymbol{dr}$ and the Matric Tensor (a 2X2 matrix) g such that $ds^2 = (du, dv)g\begin{pmatrix} du \\ dv \end{pmatrix}$

Q4. Find the length of the curves $u = 0$ and $v = \frac{\pi}{2}$

Q5. Find the surface area of the Torus.

Q6. Find the outward normal vector $\boldsymbol{n}(u, v)$ and also find the partial derivatives of this normal vector w.r.t the parameters u, v. i.e. $\boldsymbol{n}_u$and $\boldsymbol{n}_v$

Q7. Express $-\boldsymbol{n}_u$and$-\boldsymbol{n}_v$ as a combination of $\boldsymbol{r}_u$and $\boldsymbol{r}_v$ and find the Shape Operator (a 2X2 matrix) S such that $\begin{pmatrix}-\boldsymbol{n}_u\\-\boldsymbol{n}_v\end{pmatrix} = S\begin{pmatrix}\boldsymbol{r}_u\\\boldsymbol{r}_v\end{pmatrix}$.

Q8. Find $K = \det\ S$ (Gaussian Curvature of the surface) and $H = \frac{1}{2}\mathrm{tr}\ S$ (Mean Curvature of the surface). Find each at the point $u = 0, v = \frac{\pi}{2}$

Q9. Write the parametric form of the corresponding Solid Torus (filled inside)

Q10. Find the volume of the Solid Torus.