MA2073-Calculus for System Modelling-14S3-CS-Quizzes-2016/06/01

Quiz 1 What are the directions of $\frac{d\mathbf{B}}{dt}$ and $\frac{\mathbf{N}}{dt}$. Find velocity $\mathbf{v} = \frac{d\mathbf{r}}{dt}$ and acceleration $\mathbf{a} = \frac{d\mathbf{v}}{dt}$ in terms of $\mathbf{T}, \mathbf{N}, \mathbf{B}$. Also find $v \times a$ and deduce an expression for the curvature κ .

Quiz 2 Let $\mathbf{F}(x, y) = \langle 3x^2 - 2xy, 4y^2 - x^2 \rangle$. Find $\int_C \mathbf{F} \cdot d\mathbf{r}$ form (0, 0) to (1, 1) for C along y = x and $y = x^2$. Find the scalar potential ϕ of \mathbf{F} . Show that if $\mathbf{F} = \nabla \phi$ then $\int_C \mathbf{F} \cdot d\mathbf{r}$ is independent of the path.

Quiz 3 Let $\mathbf{F}(x, y) = \left\langle \frac{-y}{x^2 + y^2}, \frac{x}{x^2 + y^2} \right\rangle$. Find $\int_C \mathbf{F} \cdot d\mathbf{r}$ form (1, 0) to (-1, 0) for C along the upper and lower halves of the circle $x^2 + y^2 = 1$ and over straight lines through (1, 0), (1, 1), (-1, 1), (-1, 0). Is \mathbf{F} conservative? Is \mathbf{F} irrotational ?

Quiz 4 Let $\mathbf{F}(x,y) = \langle 3x^2 - 2xy, 4y^2 - 2xy \rangle$. Let A be the region between the curves y = x and $y = x^2$ and let C be its boundary. Find the surface area of A and verify the Greeen's theorem.

Quiz 5 Find the value of $\int_0^\infty \frac{\sin x}{x} dx$ using Fubini's theorem.

Quiz 6 Use double integrals and change of variable to find the value of $\int_0^\infty e^{-x^2} dx$.

Quiz 7 Find the value of $\iint_A (x^2 - y^2) dx dy$ where A is the region bounded by the curves y = x, y = x + 1, xy = 1, xy = 2

Quiz 8 Verify the Green's Theorem for a rectangle.

Quiz 9 Consider the vector field $\mathbf{F}(x, y, z) = \langle x^2y, 2y^2z, z^2x \rangle$. Let *C* be the curve of intersection of the surfaces $A_1 : z = x^2 + y^2$ and $A_2 : z = 2x + 3$. Verify the Stoke's theorem for the surface A_1/A_2 and also find the surface area of A_2/A_1 .

Quiz 10 Use Divergence Theorem to verify the Archimedess Principle: upward buoyant force that is exerted on a body immersed in a fluid is equal to the weight of the fluid that the body displaces.

Quiz 11 Let f = u + iv be dirrerentiable. Show that u_x, u_y, v_x, v_y exists and satisfy the CR equations: $u_x = v_y, u_y = -v_x$. Also if C is a loop in a simply connected region prove that $\oint_C f(z)dz = 0$.

Quiz 12 Let f = u + iv be analytic. *C* is a circle and *D* is a disk with center (a, b) and radius *R*. Show that $u(a, b) = \frac{1}{2\pi R} \oint_C u ds = \frac{1}{\pi R^2} \iint_D u dA$.

Quiz 13 Let f(z) be analytic in the region $R_1 < |z - a| < R_2$. Find the coefficients a_k such that $f(z) = \sum_{-\infty}^{\infty} a_k (z - a)^k$.

Quiz 14 Find the value of the integral $\int_0^{2\pi} \frac{\cos \theta}{2 + \cos \theta} d\theta$

Quiz 15 Find the value of the integral $\int_0^\infty \frac{1}{1+x^4} dx$

Quiz 16 Find the image of the lines $x, y = \cdots, -3, -1, 1, 3, \cdots$ under the function $f(z) = z^2$