## MA2073-Calculus for System Modelling-14S3-CS-Quizzes-2016/06/01

Quiz 1 What are the directions of $\frac{d \mathbf{B}}{d t}$ and $\frac{\mathbf{N}}{d t}$. Find velocity $\mathbf{v}=\frac{d \mathbf{r}}{d t}$ and acceleration $\mathbf{a}=\frac{d \mathbf{v}}{d t}$ in terms of $\mathbf{T}, \mathbf{N}, \mathbf{B}$. Also find $v \times a$ and deduce an expression for the curvature $\kappa$.

Quiz 2 Let $\mathbf{F}(x, y)=\left\langle 3 x^{2}-2 x y, 4 y^{2}-x^{2}\right\rangle$. Find $\int_{C} \mathbf{F} \cdot d \mathbf{r}$ form $(0,0)$ to $(1,1)$ for $C$ along $y=x$ and $y=x^{2}$. Find the scalar potential $\phi$ of $\mathbf{F}$. Show that if $\mathbf{F}=\nabla \phi$ then $\int_{C} \mathbf{F} \cdot d \mathbf{r}$ is independent of the path.

Quiz 3 Let $\mathbf{F}(x, y)=\left\langle\frac{-y}{x^{2}+y^{2}}, \frac{x}{x^{2}+y^{2}}\right\rangle$. Find $\int_{C} \mathbf{F} \cdot d \mathbf{r}$ form $(1,0)$ to $(-1,0)$ for $C$ along the upper and lower halves of the circle $x^{2}+y^{2}=1$ and over straight lines through $(1,0),(1,1),(-1,1),(-1,0)$. Is $\mathbf{F}$ conservative? Is $\mathbf{F}$ irrotational ?

Quiz 4 Let $\mathbf{F}(x, y)=\left\langle 3 x^{2}-2 x y, 4 y^{2}-2 x y\right\rangle$. Let $A$ be the region between the curves $y=x$ and $y=x^{2}$ and let $C$ be its boundary. Find the surface area of $A$ and verify the Greeen's theorem.

Quiz 5 Find the value of $\int_{0}^{\infty} \frac{\sin x}{x} d x$ using Fubini's theorem.
Quiz 6 Use double integrals and change of variable to find the value of $\int_{0}^{\infty} e^{-x^{2}} d x$.
Quiz 7 Find the value of $\iint_{A}\left(x^{2}-y^{2}\right) d x d y$ where $A$ is the region bounded by the curves $y=x, y=x+1, x y=1, x y=2$

Quiz 8 Verify the Green's Theorem for a rectangle.
Quiz 9 Consider the vector field $\mathbf{F}(x, y, z)=\left\langle x^{2} y, 2 y^{2} z, z^{2} x\right\rangle$. Let $C$ be the curve of intersection of the surfaces $A_{1}: z=x^{2}+y^{2}$ and $A_{2}: z=2 x+3$. Verify the Stoke's theorem for the surface $A_{1} / A_{2}$ and also find the surface area of $A_{2} / A_{1}$.

Quiz 10 Use Divergence Theorem to verify the Archimedess Principle: upward buoyant force that is exerted on a body immersed in a fluid is equal to the weight of the fluid that the body displaces.

Quiz 11 Let $f=u+i v$ be dirrerentiable. Show that $u_{x}, u_{y}, v_{x}, v_{y}$ exists and satisfy the CR equations: $u_{x}=v_{y}, u_{y}=-v_{x}$. Also if $C$ is a loop in a simply connected region prove that $\oint_{C} f(z) d z=0$.

Quiz 12 Let $f=u+i v$ be analytic. $C$ is a circle and $D$ is a disk with center $(a, b)$ and radius $R$. Show that $u(a, b)=\frac{1}{2 \pi R} \oint_{C} u d s=\frac{1}{\pi R^{2}} \iint_{D} u d A$.

Quiz 13 Let $f(z)$ be analytic in the region $R_{1}<|z-a|<R_{2}$. Find the coefficients $a_{k}$ such that $f(z)=\sum_{-\infty}^{\infty} a_{k}(z-a)^{k}$.

Quiz 14 Find the value of the integral $\int_{0}^{2 \pi} \frac{\cos \theta}{2+\cos \theta} d \theta$
Quiz 15 Find the value of the integral $\int_{0}^{\infty} \frac{1}{1+x^{4}} d x$
Quiz 16 Find the image of the lines $x, y=\cdots,-3,-1,1,3, \cdots$ under the function $f(z)=z^{2}$

