Q1. Magnetic field $\boldsymbol{B}$ induced at the origin by a curve C (given by the position vector) carrying a steady current I in the direction of the curve C is given by the Biot-Savart Law:

$$\boldsymbol{B} = \frac{\mu_0 I}{4\pi} \int_C \frac{\boldsymbol{r} \times \boldsymbol{dr}}{\|\boldsymbol{r}\|^3}$$

Here μ_0 is the permeability of the vacuum. Let the curve C be the circle with radius a and center on the z axis at a distance b from the origin and contained in a plane parallel to the xy plane. The direction of the curve is clockwise when observed from the origin. Find the magnetic field induced at the origin.

Q2. In Quantum Mechanics the wave function for the 1s orbital of the Hydrogen atom is given by

$$\psi(r, \phi, \theta) = \frac{1}{\sqrt{\pi a^3}} e^{-r/a}$$

where a is the Bohr Radius. Note that ψ^2 is a Probability Density Function (PDF) $\rho = \rho(u, v, w)$ and the probability of $u > c$ is given by $P(u > c) = \iiint_V \rho dV$ where V is the region corresponding to $u > c$. Find the probability of finding the electron outside twice the Bohr Radius, i.e. $P(r > 2a)$.

Q3. The electrostatic field $\boldsymbol{E}$ by a charge Q placed at the origin is given by $\boldsymbol{E} = \frac{Q}{4\pi\varepsilon_0 \|\boldsymbol{r}\|^3}\boldsymbol{r}$. Here ε_0 is the permittivity of the vacuum. Find the flux $\oint \boldsymbol{E} \cdot d\boldsymbol{S}$ though a sphere of radius a.

Q4. The Einstein Field Equation in General Relativity is $R_{ij} - \frac{1}{2}Rg_{ij} + \Lambda g_{ij} = \frac{8\pi G}{c^4}T_{ij}$.
Here g_{ij} are the components of the Matric Tensor(which is a symmetric Matrix) defined as

$$ds^2 = \|d\boldsymbol{r}\|^2 = (du \quad dv \quad dw)g\begin{pmatrix} du \\ dv \\ dw \end{pmatrix}$$

where $\boldsymbol{r} = \boldsymbol{r}(u, v, w)$ is the position vector. Find g for the spherical polar coordinate system (r, ϕ, θ).