Q1. If x is a solution to the following differential equation, find the other linearly independent solution using the **Wronskian Method**.

$$(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=0.$$

Q2. Use the **Frobenius Method** to solve the following **Legendre Differential Equation**. m is an integer.

$$(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+m(m+1)y=0$$

Q3. Solve the following **Heat Equation** using **Fourier Series**.

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}, 0<x<\pi, t>0$$

$$u(0,t)=0, u(\pi,t)=0, t\geq 0$$

$$u(x,0)=\begin{cases} x & ,0\leq x\leq \frac{\pi}{2} \\ \pi - x & ,\frac{\pi}{2}<x\leq \pi \end{cases}$$

Q4. Solve the following **Wave Equation** using Fourier Series.

$$\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2}, 0<x<\pi, t>0$$

$$u(0,t)=0, u(\pi,t)=0, t\geq 0$$

$$u(x,0)=\sin x, 0\leq x\leq \pi$$

$$u_t(x,0)=x^2, 0\leq x\leq \pi$$

Q5. Use **Laplace Transform** to solve the following Differential Equation. Use **Partial Fraction/Convolution/Complex Inversion Formula** to find the inverse Laplace Transform.

$$y''(x)-2y'(x)+y(x)=\sin x,\ y(0)=0, y'(0)=1.$$

Q6. Solve the following Heat Equation using Laplace Transform.

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}, 0<x<1, t>0$$

$$u(0,t)=1, u(1,t)=1, t\geq 0$$

$$u(x,0)=1+\sin \pi x, 0\leq x\leq 1$$

Q7. Solve the following **Laplace Equation** using **Fourier Transform**.

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial t^2}=0, -\infty<x<\infty, t>0$$

$$u(x,0)=f(x), -\infty<x<\infty$$

u is bounded as $t\to\infty$, u and u_x both $\to 0$ as $|x|\to\infty$

Q8. Solve following the **Airy's Differential Equation** using Fourier Transform.

$$\frac{d^2y}{\partial x^2}-xy=0$$

Solve the following time-independent **Schrödinger's Wave Equation** for the Hydrogen atom using the **Separation of Variable Method**.

$$-\frac{\hbar^2}{2m}\nabla^2\psi+V\psi=E\psi$$

You will have to solve the following differential equations(don't worry about reducing the above to the following) m, n are integers.

Q9. Associated Legendre Differential Equation: $(1-x^2)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+\left[m(m+1)-\frac{n^2}{1-x^2}\right]y=0$

Q10. Associated Leguerre Differential Equation: $x\frac{d^2y}{dx^2}+(n+1-x)\frac{dy}{dx}+my=0$