TH1. Let an orientable surface be given parametrically by $\boldsymbol{r} = \boldsymbol{r}(u, v)$. Let $\boldsymbol{n} = \boldsymbol{n}(u, v)$ be the unit outward normal vector from the convex side of the surface and the Shape matrix S be given by $\begin{pmatrix} -\boldsymbol{n}_u \\ -\boldsymbol{n}_v \end{pmatrix} = S \begin{pmatrix} \boldsymbol{r}_u \\ \boldsymbol{r}_v \end{pmatrix}$.
Now the Gaussian Curvature of the surface is defined by $K = \det S$.
Show that the Gaussian Curvature of the surface $z = f(x, y) \in \mathcal{C}^2$ is given by

$$K = \frac{f_{xx} f_{yy} - (f_{xy})^2}{(1 + f_x^2 + f_y^2)^2}$$

. Use this formula to find K for a sphere with radius R.

TH2. Let S be an orientable surface and C be its boundary which is a loop. Prove the Chandrasekhar-Wentzel lemma

$$\oint_C \boldsymbol{r} \times (d\boldsymbol{r} \times \boldsymbol{n}) = -\iint_S (\boldsymbol{r} \times \boldsymbol{n}) \nabla \cdot \boldsymbol{n} dS$$

using any method different to the one found on Wikipedia.
Also verify the above result on the upper half sphere with center at the origin and radius R.