Q1. Show that if $\boldsymbol{T}(t)$ is a unit vector then $\frac{d\boldsymbol{T}}{dt}$ is perpendicular to $\boldsymbol{T}$.

Q2. Find the curvature of a circle with radius R.

Q3. Find a scalar potential for the vector field $\boldsymbol{F} = \langle 3x^2y^2 + 2x, 2x^3y + 1 \rangle$. Also show in general that if $\boldsymbol{F} = \nabla\phi \in \mathcal{C}^1$ then $\int_C \boldsymbol{F}.d\boldsymbol{r}$ is path independent.

Q4. Evaluate the integral $\int_0^3 \int_{x^2}^9 xy^2 dydx$ as it is and after changing the order of integration.

Q5. Find the integral $\int_0^\infty e^{-x^2} dx$ by considering a double integral.

Q6. Let $S_1 : z = 2x + 3, S_2 : z = x^2 + y^2$ be two surfaces and let C be the curve on which they intersect. Verify the Stoke's Theorem on each surface for the vector field $\boldsymbol{F} =< xy, yz, zx >$.

Q7. Verify the Divergence theorem in 2D for $\boldsymbol{F} =< x^2y^2 + 2x, 2x^3y + 1 >$ on the region bounded by the curves $y = x$ and $y = x^2$.

Q8. Us the Divergence Theorem to prove the Archimedes Principle: upthrust=weight of the liquid displaced.

Q9. Use Maxwell's Equations and vector identities to show that the Electric Field in vacuum $\boldsymbol{E}$ satisfies the wawe equation: $\frac{\partial^2 \boldsymbol{E}}{\partial t^2} = c^2\nabla^2\boldsymbol{E}$ where c=speed of light in vacuum.

Q10. Express $z\overline{w}$ in terms of $\underline{z}, \underline{w}$ and find conditions for $z = x+iy$ and $w = a+ib$ to be perpendicular and parallel. Here $\underline{z} =< x, y >$ and $\underline{w} =< a, b >$.

Q11. Find the differentiable points of $z^2, |z|^2, \overline{z}$ and determine their analytic points.

Q12. Find $\oint_C \frac{z^2-2}{(z-2)^2(z-3)(z-4)} dz$ where points $2, 3$ are inside and 4 is outside of the curve c.

Q13. Find the Laurent series of the function $f(z) = \frac{1}{(z-1)(z-2)^2}$ at 2.

Q14. Find the following real integrals

$\int_0^\infty \frac{1}{1+x^4} dx$

$\int_0^\infty \frac{\sin x}{x(1+x^2)} dx$

$\int_0^\infty \frac{\sqrt{x}}{(x+1)^2} dx$