Q1. Use Spherical Polar Coordinates to write the position vector $\boldsymbol{r} = \boldsymbol{r}(\phi, \theta)$ on the surface of a sphere with centre $\mathbf{0}$ and radius R. Find the partial derivatives $\boldsymbol{r}_{\boldsymbol{\phi}}$ and $\boldsymbol{r}_{\theta}$. Also find a UNIT normal vector out from the surface $\boldsymbol{n} = \boldsymbol{n}(\phi, \theta)$ and its partial derivatives $\boldsymbol{n}_{\boldsymbol{\phi}}$ and $\boldsymbol{n}_{\theta}$.

Q2. Note that each $\boldsymbol{n}_{\boldsymbol{\phi}}$, $\boldsymbol{n}_{\theta}$ are on the plane containing $\boldsymbol{r}_{\boldsymbol{\phi}}$ and $\boldsymbol{r}_{\theta}$. So write each $\boldsymbol{n}_{\boldsymbol{\phi}}$, $\boldsymbol{n}_{\theta}$ as linear combinations of $\boldsymbol{r}_{\boldsymbol{\phi}}$ and $\boldsymbol{r}_{\theta}$ and find the 2X2 Shape Matrix S such that $\begin{pmatrix} -\boldsymbol{n}_{\boldsymbol{\phi}} \\ -\boldsymbol{n}_{\theta} \end{pmatrix} = S \begin{pmatrix} \boldsymbol{r}_{\boldsymbol{\phi}} \\ \boldsymbol{r}_{\theta} \end{pmatrix}$. Also find the Gaussian Curvature of the surface $K = \det S$.

Q3. Consider the Electric Field $\boldsymbol{E}$ at the position vector $\boldsymbol{r}$ due to a positive charge Q placed at the origin $(\boldsymbol{r} = \boldsymbol{0})$ in a medium of permittivity ε given by $\boldsymbol{E} = \frac{Q}{4\pi\varepsilon\|\boldsymbol{r}\|^3}\boldsymbol{r}$ when $\boldsymbol{r} \neq \boldsymbol{0}$. Show that $\operatorname{div} \boldsymbol{E} = 0$.

Q4. It can be shown that the Electric Flux $\Phi = \oiint_S \boldsymbol{E} \cdot d\boldsymbol{S} = \frac{Q}{\varepsilon}$ through any sphere centred at the origin. Prove that the same result holds for any closed orientable surface S that contains the origin. Also show that the Electric Flux $\Phi = \oiint_S \boldsymbol{E} \cdot d\boldsymbol{S} = 0$ through any closed orientable surface that does not contain the origin.

Q5. Prove the Archimedes principle(upthrust = weight of the liquid displayed).

Q6. Assume that the upthrust mentioned in Q5 is the resultant force in the Z direction. Now show that the resultant forces in the X and Y directions are 0 each.