## MA2023-Calculus-14S3-BM+EN+ME-Quizzes-2016/06/01

**Quiz 1** Let  $\mathbf{r}(t) = \langle a \cos t, a \sin t, ct \rangle$ . Find  $\mathbf{T}, \mathbf{N}, \mathbf{B}, \kappa, \tau$ 

**Quiz 2** Let  $\mathbf{F}(x, y) = \langle 3x^2 - 2xy, 2y^2 - x^2 \rangle$ . Find  $\int_C \mathbf{F} \cdot d\mathbf{r}$  form (0, 0) to (1, 1) for C along y = x and  $y = x^2$ . Find the scalar potential  $\phi$  of  $\mathbf{F}$ . Show that if  $\mathbf{F} = \nabla \phi$  then  $\int_C \mathbf{F} \cdot d\mathbf{r}$  is independent of the path.

**Quiz 3** Let  $\mathbf{F}(x, y) = \left\langle \frac{-y}{x^2 + y^2}, \frac{x}{x^2 + y^2} \right\rangle$ . Find  $\int_C \mathbf{F} \cdot d\mathbf{r}$  form (1, 0) to (-1, 0) for C along the upper and lower halves of the circle  $x^2 + y^2 = 1$  and over straight lines through (1, 0), (1, 1), (-1, 1), (-1, 0). Is  $\mathbf{F}$  conservative? Is  $\mathbf{F}$  irrotational ?

**Quiz 4** Let  $\mathbf{F}(x, y) = \langle 3x^2 - 2xy, 2y^2 - 2xy \rangle$ . Verify the Green's theorem for the region bounded by y = x and  $y = x^2$ .

**Quiz 5** Use double integrals and change of variable to find the value of  $\int_0^\infty e^{-x^2} dx$ .

**Quiz 6** Consider the vector field  $\mathbf{F}(x, y, z) = \langle x^2y, y^2z, z^2x \rangle$ . Let *C* be the curve of intersection of the surfaces  $A_1 : z = x^2 + y^2$  and  $A_2 : z = 2x + 3$ . Verify the Stoke's theorem for the surface  $A_1/A_2$  and also find the surface area of  $A_2/A_1$ .

**Quiz 7** Consider the vector field  $\mathbf{F}(x, y, z) = \langle x^2 y, y^2 z, z^2 x \rangle$ . Let V be the volume bounded by the surfaces  $z = x^2 + y^2$  and z = 2x + 3. Verify the Divergence theorem. Also find the volume of V.

**Quiz 8** Is  $\langle \mathbb{R}, \cdot, + \rangle$  a field? How to convert  $\mathbb{R}^2 = \{(x, y) | x, y \in \mathbb{R}\}$  to a field?

**Quiz 9** Let f = u + iv be dirrerentiable. Show that  $u_x, u_y, v_x, v_y$  exists and satisfy the CR equations:  $u_x = v_y, u_y = -v_x$ . Also if C is a loop in a simply connected region prove that  $\oint_C f(z)dz = 0$ .

**Quiz 10** Let  $f(z) = \frac{z}{(z-1)^2(z-2)}$ . Find constants  $a_k$  such that  $f(z) = \sum_{k=-\infty}^{\infty} a_k (z-1)^k$ .

**Quiz 11** Find  $\int_0^\infty \frac{1}{1+x^4} dx$ 

Quiz 12 Find  $\mathcal{L}^{-1}\left\{\frac{s}{(s-1)^2(s-2)}\right\}$ .

**Quiz 13** Find the image of the lines  $x, y = \cdots, -3, -1, 1, 3, \cdots$  under the function  $f(z) = z^2$