Q1. Consider the Legendre ODE: $(1-x^2)y'' - 2xy' + 2y = 0$. If x is a solution, find the other linearly independent solution.

Q2. Find the Legendre Series for $|x|$ upto 10 terms.

Q3. Solve the Non-homogeneous Associated Legendre ODE:
$(1-x^2)y'' - 2xy' + (2 - \frac{1}{1-x^2})y = x$.

Q4. Solve the Bessel ODE: $x^2y'' + xy' + (x^2 - \nu^2)y = 0$ for $\nu = 0$.

Q5. Derive a formula for the Bessel series of the from $f(x) = \sum_{k=0}^{\infty} a_k J_\nu(\lambda_k x)$ and find two terms of the series for $f(x) = x$. Take $a = \nu = 1$.

Q6. Use Laplace Transform to solve the ODE: $y'' + 5y' + 6y = \cos x$,
$y(0) = 1, y'(0) = 2$.

Q7. Use Laplace Transform to solve the PDE: $\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2} + \sin x$,
$y(0,t) = 0, y(\pi,t) = 0, y(x,0) = 0, \frac{\partial y}{\partial t}(x,0) = 0$

Q8. Find the Fourier Series for x^2 and deduce the value of $\sum_{n=1}^{\infty} \frac{1}{n^2}$

Q9. Solve the Heat equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$
$u(0,t) = 0, u(\pi,t) = 0; t \geq 0 \qquad u(x,0) = x(\pi - x); 0 \leq x \leq \pi$.
Use separation of variables(see why $\mu \geq 0$ is not possible) and Fourier series(see how the domain of f can be extended for $[-\pi, 0]$.

Q10. Solve the Wave equation $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$
$u(0,t) = 0, u(\pi,t) = 0; t \geq 0 \qquad u(x,0) = \sin x, \frac{\partial u}{\partial t}(x,0) = x^2; 0 \leq x \leq \pi$.
Use separation of variables and Fourier series.

Q11. Use Complex Inversion for Laplace Transform to find the Inverse Laplace Transform of $\frac{s}{(s+1)^2(s-1)}$. Verify the answer by a different method for Inverse Laplace Transform.

Q12. Prove $\int_{-\infty}^{\infty} |f(x)|^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} |\mathcal{F}f(x)|^2 dx$ or
$\int_{-\infty}^{\infty} \mathcal{F}f(x)g(x)dx = \int_{-\infty}^{\infty} f(x)\mathcal{F}g(x)dx$ if $f, g \in \mathcal{S}$.

Q13. Use Fourier Transform to Solve the Laplace's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$
$u(x,0) = f(x) \qquad u(x,y)$ bounded as $y \to \infty$.
Also compute the answer for $f(x) = x$ for $x > 0$ and 0 otherwise.

Q14. Find the range of y for which $yu_{xx} + u_{yy} = 0$ is Hyperbolic. Reduce the PDE to its canonical form for Hyperbolic PDEs(only the u_{st} remaining in the 2nd order terms w.r.t. the new variables s, t).