Q1. Use power series on BOTH sides of the ODE: $y'' - xy' + y = 1$ to find the solution in the form $y = a_1 u(x) + a_2 v(x) + z(x)$. DO NOT assume any solution or use Wronskian. Here a_1, a_2 are coefficients of the power series expansion of y at 0.

Q2. In Q1 we found the solution to $y'' - xy' + y = 1$ in the form $y = a_1 u(x) + a_2 v(x) + z(x)$.
Use Wronskian to express $v(x)$ in terms of the erfi function and other well-known functions up to a constant.
Here $\mathrm{erfi}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{t^2} dt$.

Q3. Consider the equivalent ODE: $f''(t) - tf'(t) + f(t) = 1$. Find $F(s)$, the Laplace Transform of the solution. DO NOT find the Inverse Laplace Transform. Use the result $\mathcal{L}\big(tf(t)\big)(s) = -F'(s)$ where $F(s) = \mathcal{L}\big(f(t)\big)(s)$.