

```
In[1]:= DSolve[(1 - x^2) y''[x] - 2 x y'[x] + n (n + 1) y[x] == 0, y[x], x]
```

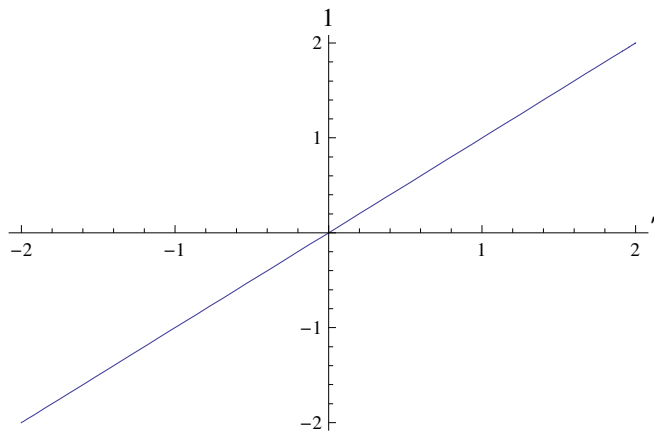
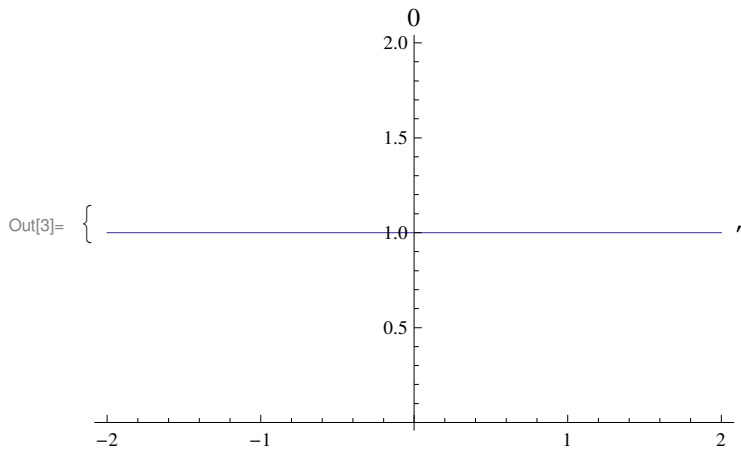
```
Out[1]= {{y[x] -> C[1] LegendreP[n, x] + C[2] LegendreQ[n, x]}}
```

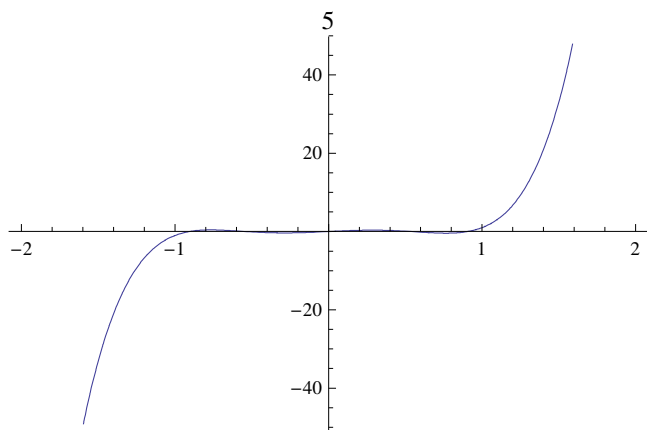
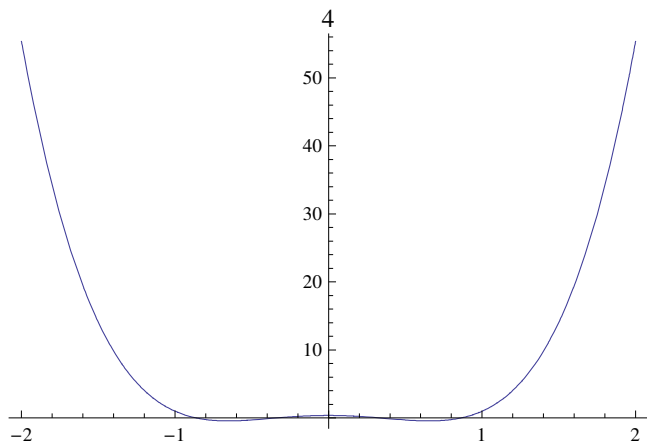
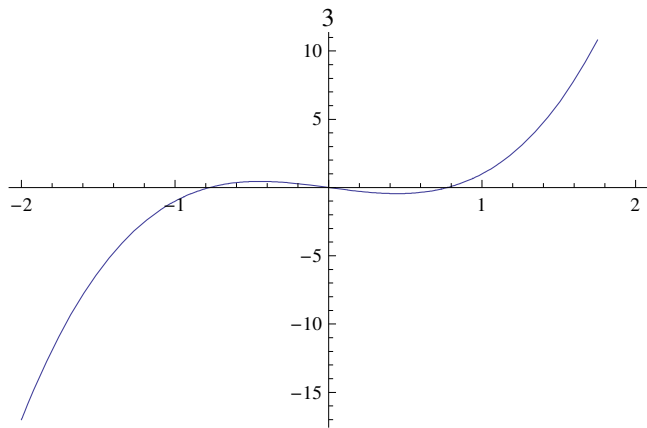
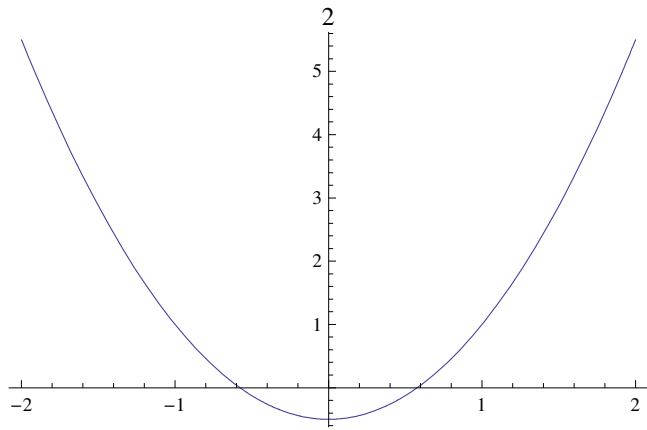
```
In[2]:= Table[{n, LegendreP[n, x]}, {n, 0, 5}] // TableForm
```

```
Out[2]/TableForm=
```

```
0  1
1  x
2  1/2 (-1 + 3 x^2)
3  1/2 (-3 x + 5 x^3)
4  1/8 (3 - 30 x^2 + 35 x^4)
5  1/8 (15 x - 70 x^3 + 63 x^5)
```

```
In[3]:= Table[Plot[LegendreP[n, x], {x, -2, 2}, PlotLabel -> n], {n, 0, 5}]
```



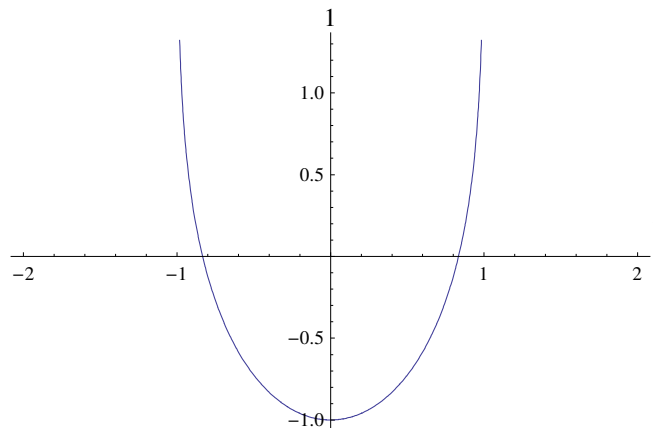
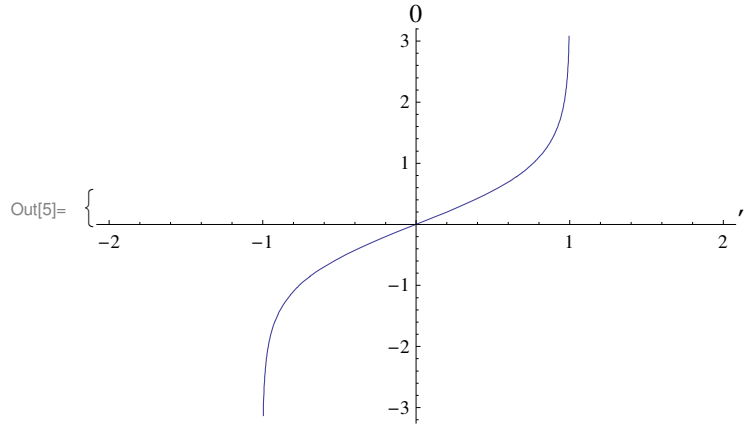


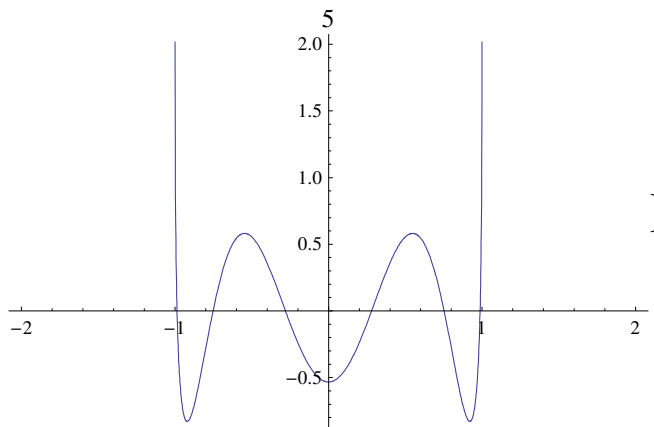
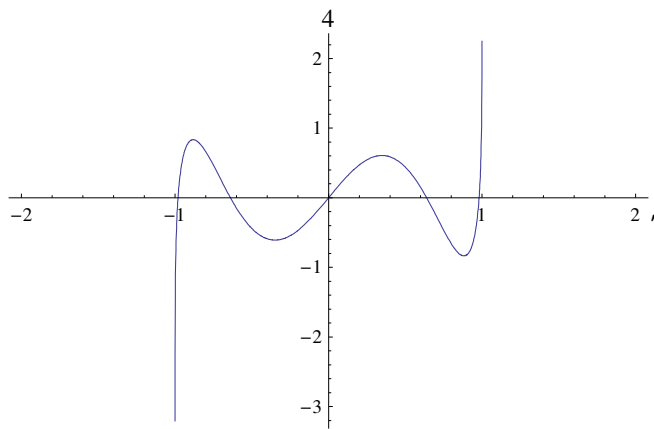
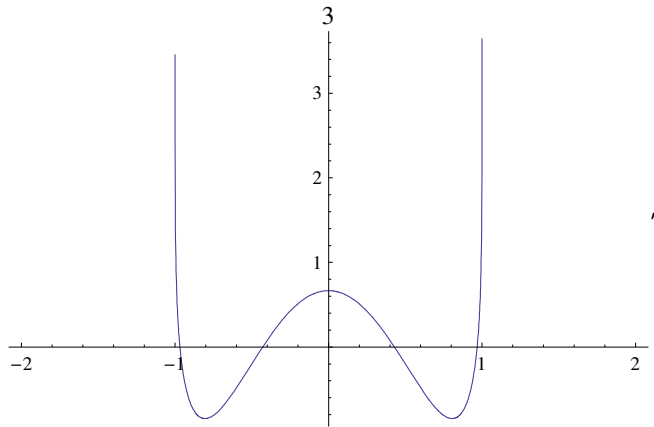
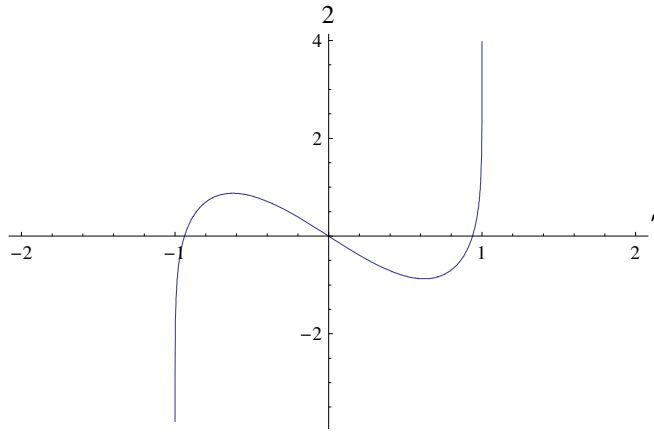
```
In[4]:= Table[{n, LegendreQ[n, x]}, {n, 0, 5}] // TableForm
```

Out[4]/TableForm=

- 0 $-\frac{1}{2} \text{Log}[1 - x] + \frac{1}{2} \text{Log}[1 + x]$
- 1 $-1 + x \left(-\frac{1}{2} \text{Log}[1 - x] + \frac{1}{2} \text{Log}[1 + x]\right)$
- 2 $-\frac{3x}{2} + \frac{1}{2} (-1 + 3x^2) \left(-\frac{1}{2} \text{Log}[1 - x] + \frac{1}{2} \text{Log}[1 + x]\right)$
- 3 $\frac{2}{3} - \frac{5x^2}{2} - \frac{1}{2} x (3 - 5x^2) \left(-\frac{1}{2} \text{Log}[1 - x] + \frac{1}{2} \text{Log}[1 + x]\right)$
- 4 $\frac{55x}{24} - \frac{35x^3}{8} + \frac{1}{8} (3 - 30x^2 + 35x^4) \left(-\frac{1}{2} \text{Log}[1 - x] + \frac{1}{2} \text{Log}[1 + x]\right)$
- 5 $-\frac{8}{15} + \frac{49x^2}{8} - \frac{63x^4}{8} + \frac{1}{8} x (15 - 70x^2 + 63x^4) \left(-\frac{1}{2} \text{Log}[1 - x] + \frac{1}{2} \text{Log}[1 + x]\right)$

In[5]:= `Table[Plot[LegendreQ[n, x], {x, -2, 2}, PlotLabel -> n], {n, 0, 5}]`





$$\text{In[6]:= } a[k_] := \frac{2k+1}{2} \int_{-1}^1 \text{LegendreP}[k, x] \text{Abs}[x] dx$$

```
In[7]:= A = Table[a[k], {k, 0, 20}]
```

$$\text{Out[7]} = \left\{ \frac{1}{2}, 0, \frac{5}{8}, 0, -\frac{3}{16}, 0, \frac{13}{128}, 0, -\frac{17}{256}, 0, \frac{49}{1024}, \right. \\ \left. 0, -\frac{75}{2048}, 0, \frac{957}{32768}, 0, -\frac{1573}{65536}, 0, \frac{5291}{262144}, 0, -\frac{9061}{524288} \right\}$$

```
In[8]:= g[n_, x_] := Sum[A[[k + 1]] LegendreP[k, x], {k, 0, n}]
```

```
In[9]:= Table[Plot[{Abs[x], g[n, x]}, {x, -1, 1}, PlotLabel -> n], {n, 0, 20}]
```

