

```
In[1]:= DSolve[x^2 y''[x] + x y'[x] + (x^2 - n^2) y[x] == 0, y[x], x]
```

```
Out[1]:= {{y[x] -> BesselJ[n, x] C[1] + BesselY[n, x] C[2]}}
```

```
In[2]:= Table[{n, BesselJ[n, x]}, {n, 0, 5}] // TableForm
```

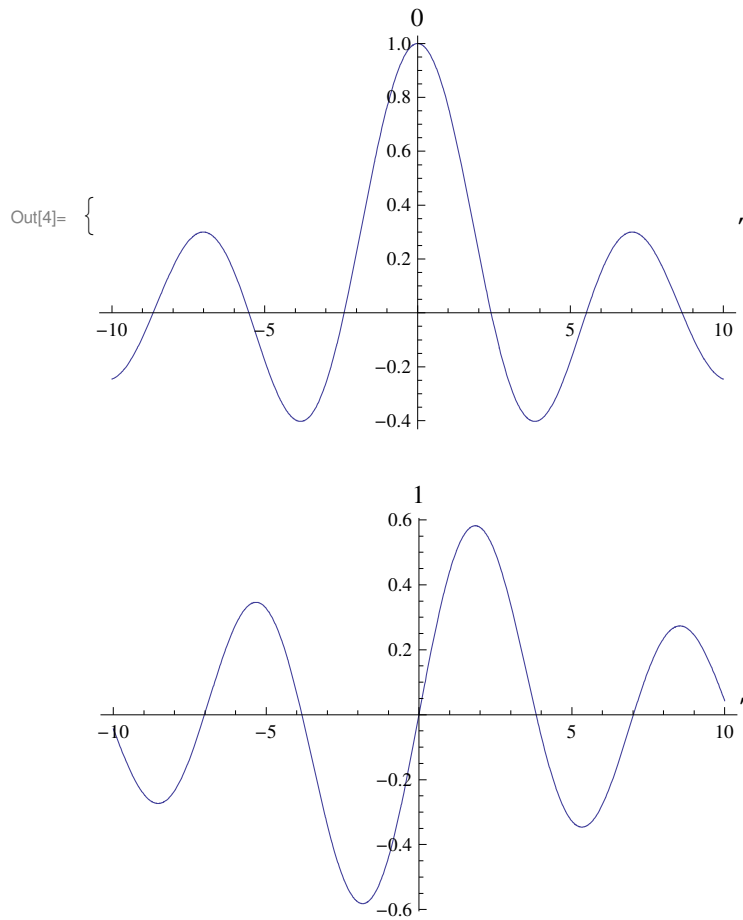
```
Out[2]/TableForm=
```

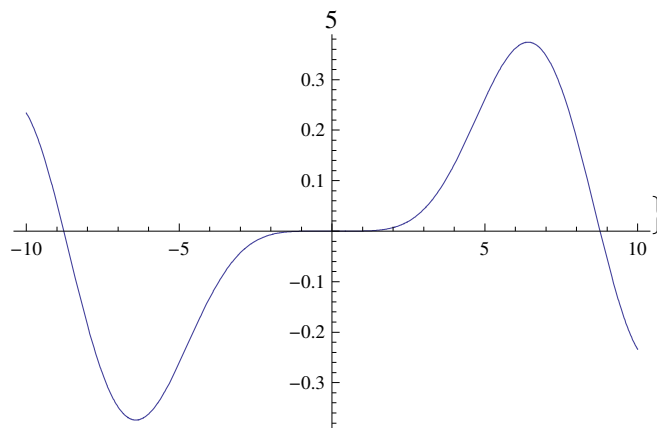
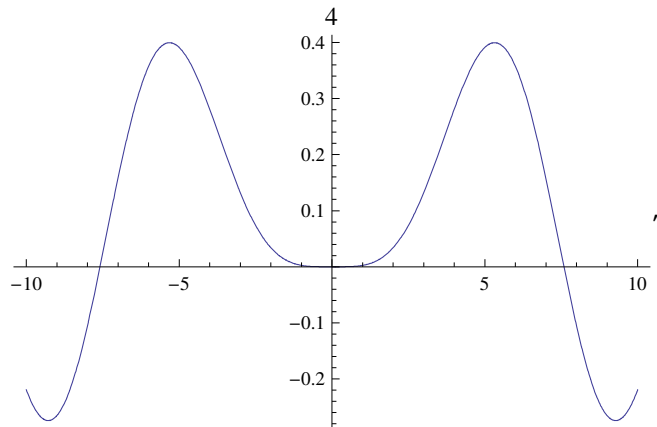
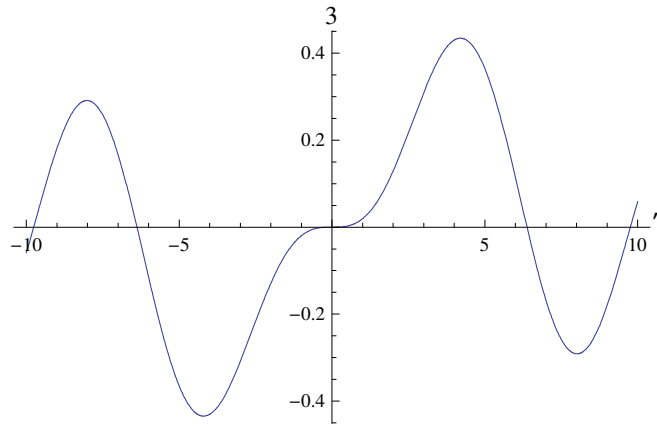
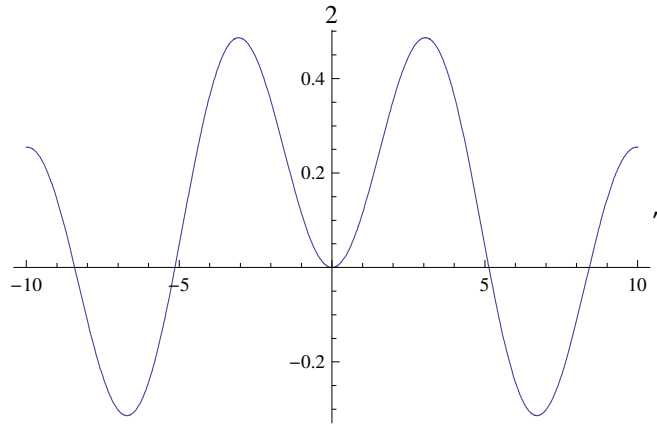
```
0 BesselJ[0, x]
1 BesselJ[1, x]
2 BesselJ[2, x]
3 BesselJ[3, x]
4 BesselJ[4, x]
5 BesselJ[5, x]
```

```
In[3]:= Series[BesselJ[1, x], {x, 0, 10}]
```

```
Out[3]=  $\frac{x}{2} - \frac{x^3}{16} + \frac{x^5}{384} - \frac{x^7}{18432} + \frac{x^9}{1474560} + O[x]^{11}$ 
```

```
In[4]:= Table[Plot[BesselJ[n, x], {x, -10, 10}, PlotLabel -> n], {n, 0, 5}]
```





```
In[5]:= Table[{n, BesselY[n, x]}, {n, 0, 5}] // TableForm
```

Out[5]/TableForm=

- 0 BesselY[0, x]
- 1 BesselY[1, x]
- 2 BesselY[2, x]
- 3 BesselY[3, x]
- 4 BesselY[4, x]
- 5 BesselY[5, x]

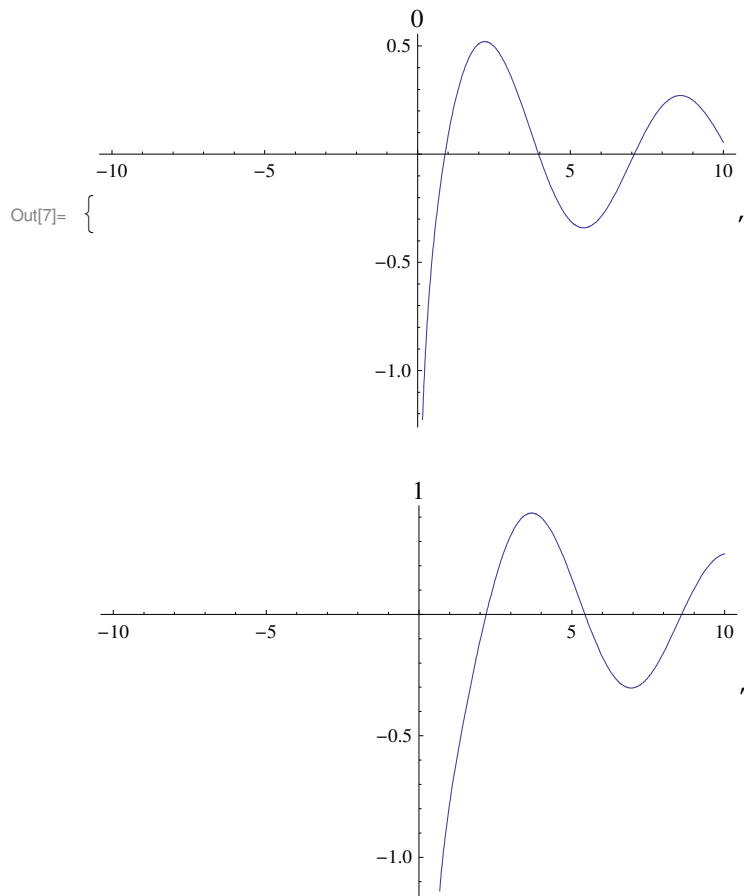
In[6]:= Series[BesselY[1, x], {x, 0, 10}]

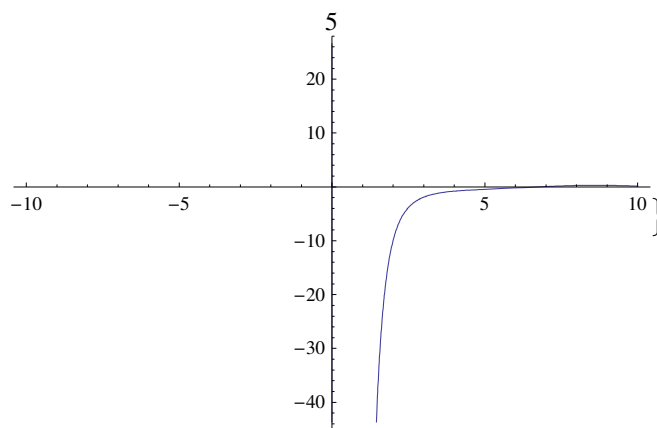
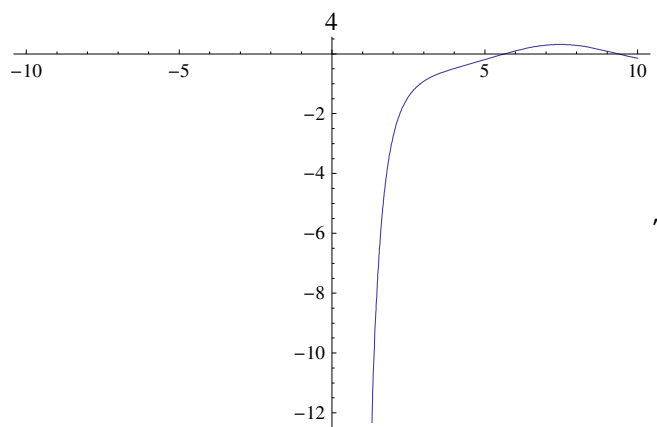
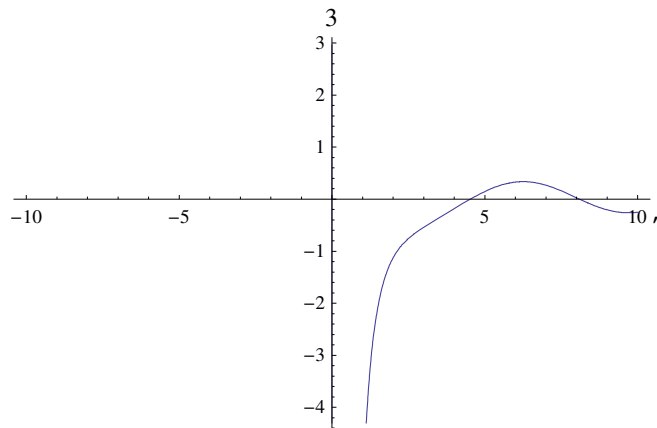
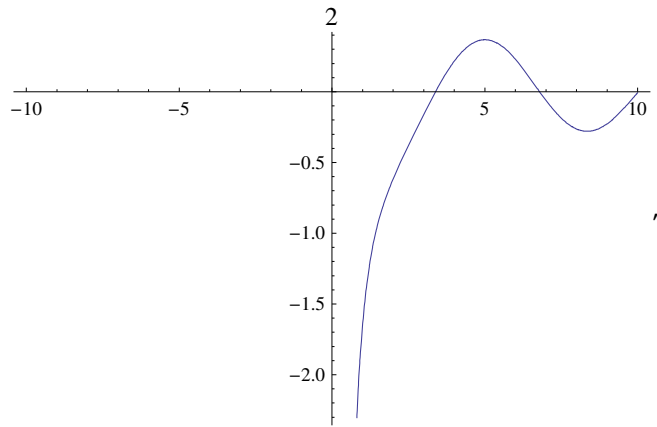
$$\text{Out[6]} = -\frac{2}{\pi x} + \frac{(-1 + 2 \text{EulerGamma} - 2 \text{Log}[2] + 2 \text{Log}[x]) x}{2 \pi} + \frac{(5 - 4 \text{EulerGamma} + 4 \text{Log}[2] - 4 \text{Log}[x]) x^3}{32 \pi} +$$

$$\frac{(-5 + 3 \text{EulerGamma} - 3 \text{Log}[2] + 3 \text{Log}[x]) x^5}{576 \pi} + \frac{(47 - 24 \text{EulerGamma} + 24 \text{Log}[2] - 24 \text{Log}[x]) x^7}{221184 \pi} +$$

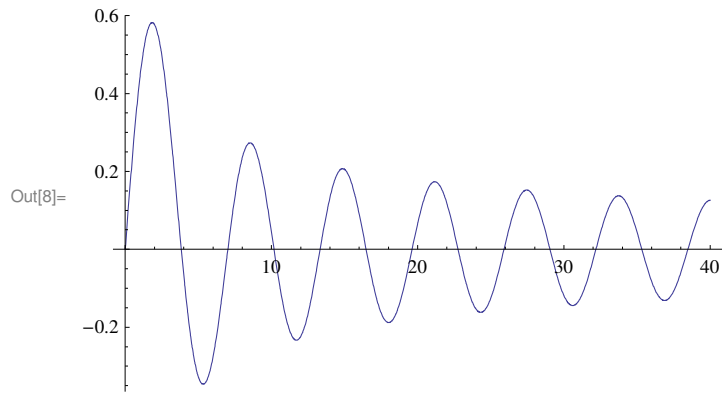
$$\left(-\frac{131}{44236800 \pi} + \frac{\text{EulerGamma}}{737280 \pi} - \frac{\text{Log}[2]}{737280 \pi} + \frac{\text{Log}[x]}{737280 \pi} \right) x^9 + O[x]^{11}$$

In[7]:= Table[Plot[BesselY[n, x], {x, -10, 10}, PlotLabel -> n], {n, 0, 5}]





```
In[8]:= Plot[BesselJ[1, x], {x, 0, 40}]
```



```
In[9]:= T = Table[FindRoot[BesselJ[1, x] == 0, {x, n}, WorkingPrecision -> 50][[1]][[2]],
  {n, {4, 7, 10, 13, 16, 20, 23, 27, 29, 32}}]
```

```
Out[9]= {3.8317059702075123156144358863081607665645452742878,
  7.0155866698156187535370499814765247432763115029039,
  10.173468135062722077185711776775844069819512500192,
  13.323691936314223032393684126947876751216644731358,
  16.470630050877632812552460470989551449438126822273,
  19.615858510468242021125065884137509850247402661881,
  22.760084380592771898053005152182257592905370738073,
  25.903672087618382625495855445979874287905427031367,
  29.046828534916855066647819883531961100414171793084,
  32.189679910974403626622984104460369219052867711015}
```

$$\text{In[10]:= } a[k_] := \frac{2}{\text{BesselJ}[2, T[[k]]]^2} \int_0^1 x \text{BesselJ}[1, T[[k]] x] x dx$$

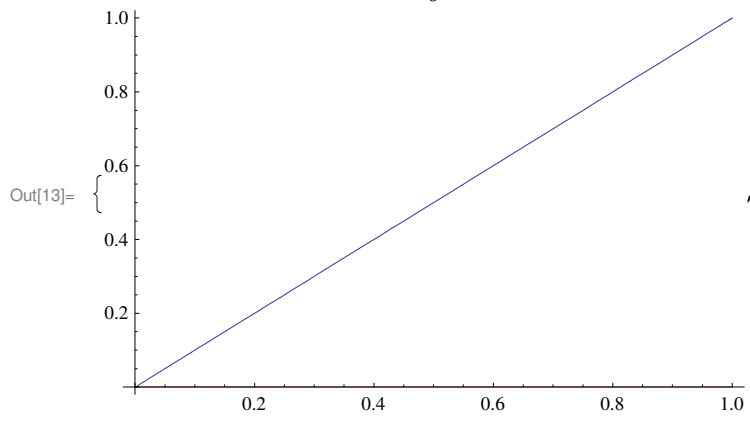
```
In[11]:= A = Table[a[k], {k, 1, 10}]
```

```
Out[11]= {1.29596161810896476962071313341474466764347075295,
  -0.949898517979951699560388598018898388625314662,
  0.7872885617253092918605672391519512878821772, -0.687437954229748454294478277894166101963302,
  0.61806445705099720820497920954034829887282, -0.566235753779818373601093597079759503283,
  0.52560544562030406601963227000343645792, -0.492640850692782610205150997631860211,
  0.46519705754835185379886751013326412, -0.4418858627311921174190179074366278}
```

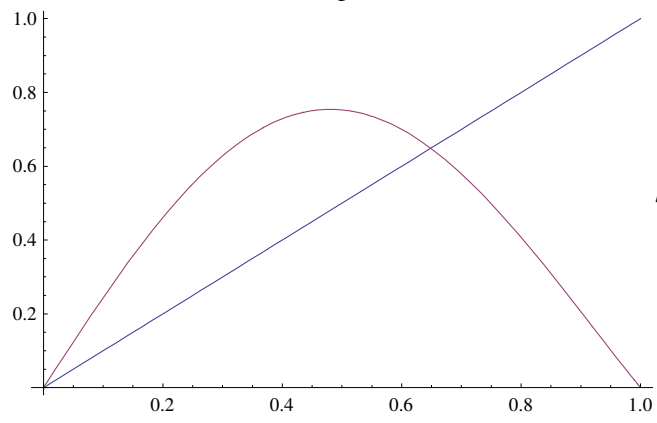
$$\text{In[12]:= } g[x_, n_] := \sum_{k=1}^n A[[k]] \text{BesselJ}[1, T[[k]] x]$$

```
In[13]:= Table[Plot[{x, g[x, n]}, {x, 0, 1}, PlotLabel -> n], {n, 0, 10}]
```

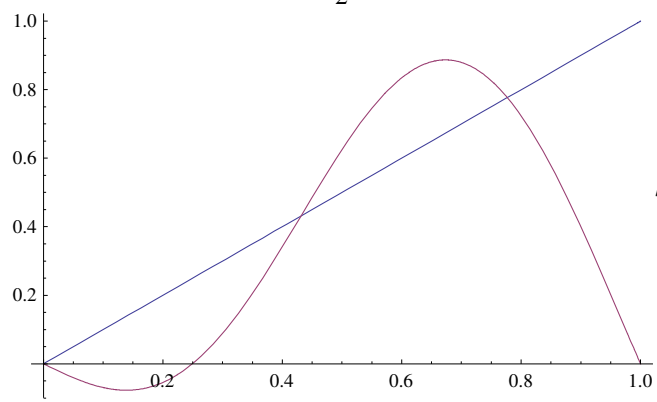
0

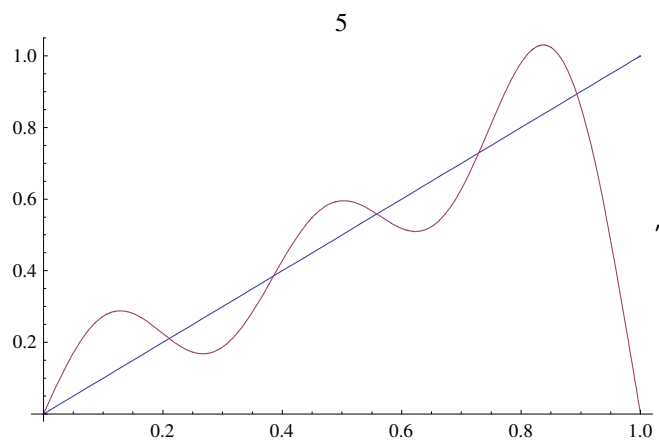
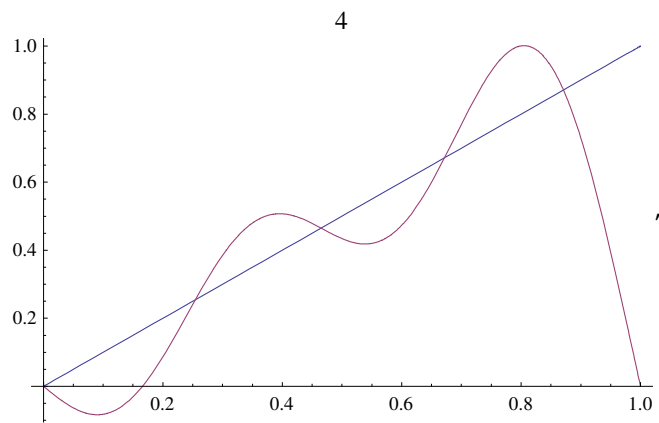
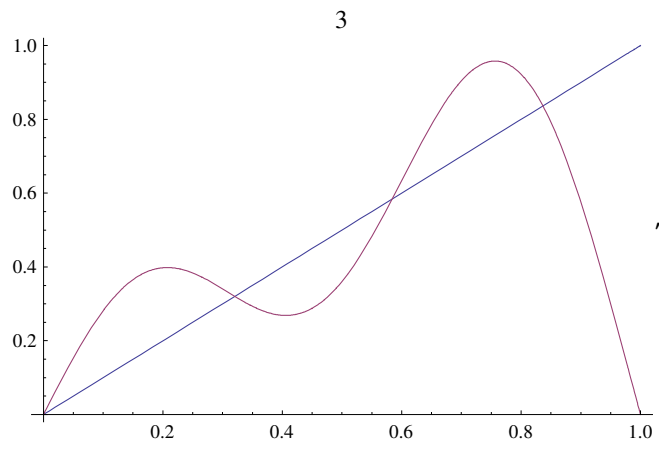


1

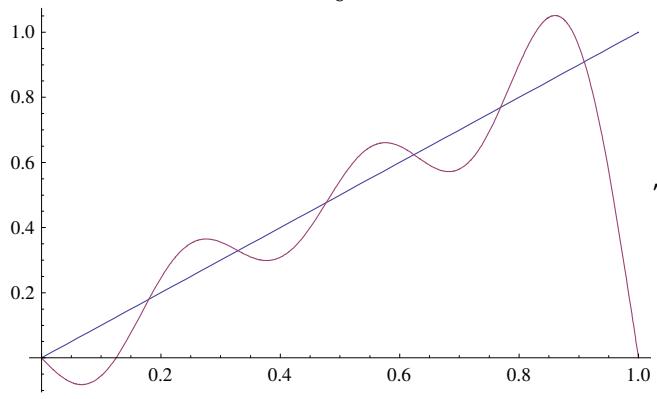


2

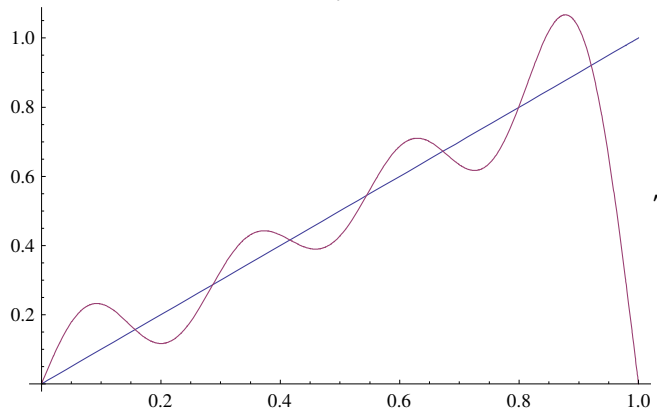




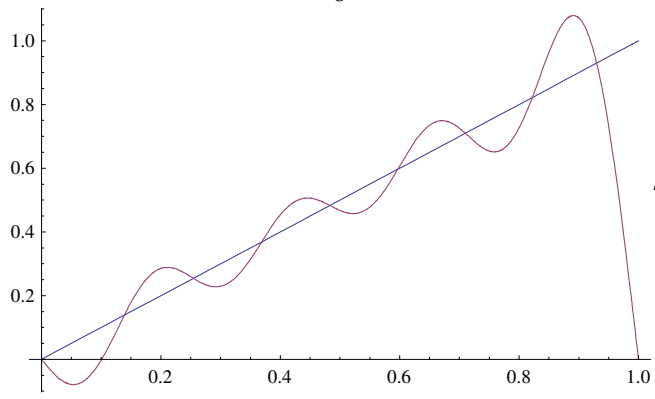
6



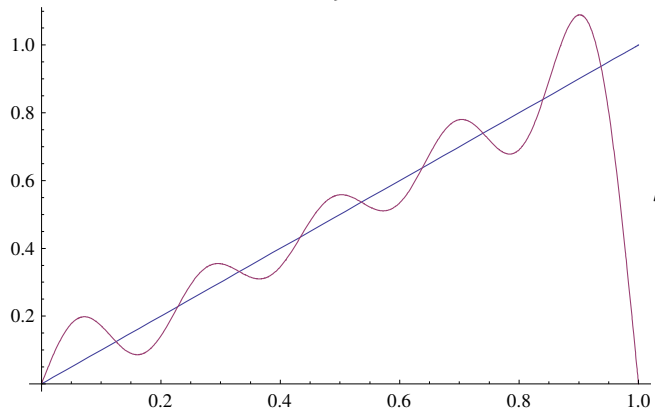
7



8



9



10

