Q1. Use the Frobenius Method to solve the following Legendre Differential Equation. n is an integer.
$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0$$

Q2. Solve $y'' - 2y' + y = \sin x, y(0) = 0, y'(0) = 0$ using Laplace Transform.

Q3. Solve the following Heat Equation
$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, 0 < x < 1, t > 0$$
$$u(0,t) = 1, u(1,t) = 1, t \geq 0$$
$$u(x,0) = 1 + \sin\pi x, 0 \leq x \leq 1$$
Use Laplace transform on the variable t and then the Wronskian Method.

Q4. Use Laplace transform on the variable t and then on the variable x to solve the above Heat Equation.

Q5. Find the Legendre Series of $\sin \pi x$.

Q6. Find the Fourier Series for x^2 and deduce the value of $\sum_{n=1}^{\infty}\frac{1}{n^2}$.

Q7. Solve the following Heat Equation.
$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, 0 < x < \pi, t > 0$$
$$u(0,t) = 0, u(\pi,t) = 0, t \geq 0$$
$$u(x,0) = \begin{cases} x & , 0 \leq x \leq \frac{\pi}{2} \\ \pi - x & , \frac{\pi}{2} < x \leq \pi \end{cases}$$
Use Separation of Variables and then the Fourier Series.

Q8. Let $T_N(x) = \sum_{n=-N}^{N} b_n\, e^{-inx}$ and let the 2-Norm defined by $\|f\| = \sqrt{\langle f, f\rangle}$
where $\langle f, g\rangle = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)g(x)dx$.
Show that the distance between f and T_N defined by $\|f - T_N\|$ is minimized if
$b_n = c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)e^{inx}dx$, ie. If T_N is the finite Fourier Series of f defined by $S_N(x) = \sum_{n=-N}^{N} c_n\, e^{-inx}$.

Q9. Use the Parseval's Identity $\|f\|^2 = \sum_{n=-\infty}^{\infty}|c_n|^2$ to find the value of $\sum_{n=1}^{\infty}\frac{1}{n^4}$.

Q10. Solve the Airy's ODE $y'' - xy = 0$ using Fourier Transform.

Q11. Solve the first order PDE $xu_x - yu_y + y^2u = y^2$ using Variable Transformation and Method of Characteristics.

Q12. Use the Complex Inversion Formula $f(t) = \mathcal{L}^{-1}\{\hat{f}(z)\} = \sum \mathrm{Res}[\hat{f}(z)e^{zt}]$ to find $\mathcal{L}^{-1}\left\{\frac{1}{z(z-1)^2}\right\}$.

Q13. (MID redo)Use Fourier Series to solve the following Wave Equation.
$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, 0 < x < \pi, t > 0$$
$$u(0,t) = 0,\ \ u(\pi,t) = 0,\ \ u(x,0) = \sin x,\ \ \frac{\partial u}{\partial t}(x,0) = x^2$$

Q14. (MID makeup-NO need to do)Use any method of your choice to solve the following Equation.
$$\frac{\partial^2 u}{\partial t^2} + \frac{\partial^2 u}{\partial x^2} = u, 0 < x < \pi, t > 0$$
$$u(0,t) = 0,\ \ u(\pi,t) = 0,\ \ u(x,0) = \sin x,\ \ \frac{\partial u}{\partial t}(x,0) = x^2$$