Q2. Find the value of $\ln(0.2)$ accurate to 10^{-6}.

Q3. Find the Lagrange Polynomial for the data set $\{(2,1),(3,2),(4,3),(6,4)\}$

Q4. Let $f(x) = 4\sin^2(\frac{\pi x}{12})$ and $p_n(x)$ be its n th degree Lagrange polynomial on $[0,6]$ for equispaced points. Find n such that $|f(x) - p_n(x)| < 10^{-6}$.

Q5. Gamma function is defined by $\Gamma(x) = \int_0^{\inf} e^{-t}t^{x-1}dt$. Show that $\Gamma(x)$ exists for $x > 0$.

Q6. Find a, b such that $\nabla E = 0$. Show that the solution corresponds to a global minimum.

Q7. For the data set $\{(x_k, y_k)\}, k = 0, 1, , n$ a natural cubic spline is a twice differentiable piecewise cubic polynomial $p(x)$ which satisfies $p(x_k) = y_k$ with $p''(x_0) = p''(x_n) = 0$. Let $p(x) = \sum_{k=0}^{n-1} p_k(x)$ where $p_k(x)$ is the part of $p(x)$ on $[x_k, x_{k+1}]$ which is 0 elsewhere. Assume that $p_k(x) = a_k(x-x_k)^3 + b_k(x-x_k)^2 + c_k(x-x_k) + d_k$ and that $x_{k+1} - x_k = h$ is a constant. With $s_k = p''(x_k)$ derive the formula $s_{k+2} + 4s_{k+1} + s_k = \frac{6}{h^2}(y_{k+2} - 2y_{k+1} + y_k), k = 0, 1, , n-2$. Also write the system of equations in matrix form that must be solved to find s_k for $k = 1, , n-1$. Get the value of USD w.r.t. LKR for the 1st day of every month this year and predict the value for the 1st November this year using natural cubic splines (just show the data set, the calculated s_k values and the final answer).

Q8. Let n be even. If $p_0(x)$ is the 2nd degree Lagrange polynomial of f on $[x_0, x_2]$ prove that $\int_{x_0}^{x_1} p_0(x)dx = \frac{h}{3}[f(x_0) + 4f(x_1) + f(x_2)]$ and derive an error formula on $[x_0, x_2]$. Also find $S(1) = \int_0^2 \sin(x^2)dx$ accurate to 10^{-3}.

Q9. Directly show that the Simpson's rule integrates cubics exactly.

Q10. One method of doing numerical integration is Gaussian Quadrature. Note that both the Trapezoidal and the Simpsons rules looks like $\int_a^b f(x)dx \approx \sum_k w_k f(x_k)$ and we knew x_k and found w_k. In this method we find both x_k and w_k so that the integral and the sum are equal for a given n degree polynomial $p(x)$. It is achieved by forcing both sides equal for each power of x^j for $j = 0, 1, 2, , n$. What is the degree of the polynomial we need to use if we want 3 points and the corresponding 3 weights? Find them for $[a, b] = [-1, 1]$ and use it to approximate $\int_0^1 \sin(x^2)dx$.

Q11. Use the Fixed Point Method to find the root of $f(x) = x - e^{-x}$.

Q12. Derive a method to find solutions to the system of non-linear equations $f(x, y) = 0$ and $g(x, y) = 0$ of two variables. Use it to find the solution to the system of non-linear equations that you had to solve in Quiz 3 on Gaussian Quadrature.

Note: For $f(x) = 0$ using Taylor series with $x \approx x_{k+1} = x_k + h$ we have $0 \approx f(x_{k+1}) = f(x_k + h) \approx f(x_k) + hf'(x_k)$, i.e. $h = x_{k+1} - x_k \approx -\frac{f(x_k)}{f'(x_k)}$. We define $x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$. In this example use two variable Taylor series for $f(x, y)$ and $g(x, y)$ with $x_{k+1} = x_k + h$ and $y_{k+1} = y_k + \ell$. Write the code in your preferred language, include your name, index number and field as a comment in the code. Print the code and the output and handover to your tutor on or before 24/11/2017, highlight your information. Please see 2DNewton.pdf for a sample code in MATHEMATICA.

Q13. One method to find the maximum of a multivariate function $f(x, y)$ is called the Steepest Descent Method. Here we start at a given point (a_0, b_0) and select the direction of the maximum slope at (a_0, b_0). Then we follow that maximum slope direction till we get the maximum along that direction as a one variable function, say at (a_1, b_1) and we repeat the process. Show that the maximum directions at (a_0, b_0) and (a_1, b_1) are perpendicular. If $f(x, y) = x^3 + 3xy^2 - 75x - 9y^2$, write the first two steps of the Steepest Descend Method starting from $(0, 0)$. Write a code and find the point we get.