Quiz 1. *Write $x^5 - x - 1 = 0$ as $x = g(x)$ in 6 different ways to be solved by the iterative method. Which methods converge?*

Quiz 2. *Let $\{x_k\}_{k=0}^{\infty}$ be the sequence generated according to $x_{k+1} = g(x_k)$ in the iterative method. Show that $|x_m - x_n| \leq \frac{L^m}{1-L}|x_1 - x_0|$.*

Quiz 3. *Consider the iterative method $x = (1+x)^{1/5} = g(x)$ for finding the real roots of $x^5 - x - 1 = 0$ in $[1, 2]$. Find the no. of iterations need to find the root to an accuracy of 0.001 and that root.*

Quiz 4. *Let $f, g \in \mathcal{C}$ and g does not change sign. Then show that there exists $\zeta \in (a, b)$ such that $\int_a^b f(x)g(x)dx = f(\zeta)\int_a^b g(x)dx$.*
Let $f \in \mathcal{C}^{n+1}$. Deduce that $\frac{1}{n!}\int_a^x f^{(n+1)}(t)(x-t)^n dt = \frac{f^{(n+1)}(\zeta)}{(n+1)!}(x-a)^{n+1}$ where ζ is between x and a.

Quiz 5. *Let $f \in \mathcal{C}^{n+1}$. Prove by induction that $f(x) = \sum_{k=0}^{n} \frac{f^{(k)}(a)}{k!}(x-a)^k + \frac{1}{n!}\int_a^x f^{(n+1)}(t)(x-t)^n dt$*

Quiz 6. *Suppose we want to find the roots of $2x - \cos x = 0$ and $[0, 1]$ contains a root. For each of the methods Bisection/Iterative/Newton's, find the no. of iterations needed to find the root to an accuracy of 0.001 starting from a suitable x_0 and find the root by each of these methods.*

Quiz 7. *Suppose we want to find the roots of $f(x) = \tan^{-1} x = 0$, using the Newton's method, starting from x_0. Find the value of z such that the method enters into a cycle for $z = x_0$.*

Quiz 8. *Find the number of intervals needed to find the value of $\int_0^1 e^{-\frac{x^2}{2}}dx$ accurate to 0.001 using the Trapezoidal rule and find the given integral to that accuracy.*

Quiz 9. *Given that $\int_a^\infty |f(x)|dx$ is converging, prove that $\int_a^\infty f(x)dx$ is converging. Use the above result to prove that $\int_0^\infty \sin(x^2)dx$ is converging.*

Quiz 10. *Derive the formula for the Simpson's rule, which approximate the function by a parabola over two intervals.*

Quiz 11. *Derive an error formula for the Simpson's rule and find the value of $\int_{-1}^1 e^{-\frac{x^2}{2}}dx$ accurate to 0.001.*

Quiz 12. *For the data set $\{(x_k, y_k)\}, k = 0, 1, , n$ a natural cubic spline is a twice differentiable piecewise cubic polynomial $p(x)$ which satisfies $p(x_k) = y_k$ with $p''(x_0) = p''(x_n) = 0$. Let $p(x) = \sum_{k=0}^{n-1} p_k(x)$ where $p_k(x)$ is the part of $p(x)$ on $[x_k, x_{k+1}]$ which is 0 elsewhere. Assume that $p_k(x) = a_k(x-x_k)^3 + b_k(x-x_k)^2 + c_k(x-x_k) + d_k$ and that $x_{k+1} - x_k = h$ is a constant. With $s_k = p''(x_k)$ derive the formula $s_{k+2} + 4s_{k+1} + s_k = \frac{6}{h^2}h^2\left(y_{k+2} - 2y_{k+1} + y_k\right), k = 0, 1, , n-2$. Also write the system of equations in matrix form that must be solved find s_k for $k = 1, , n-1$. Get the value of USD w.r.t. LKR for the 1st day of every month this year and predict the value for the 1st February next year using natural cubic splines (just show the data set, the calculated s_k values and the final answer).*

Quiz 13. *One method of doing numerical integration is Gaussian Quadrature. Note that both the Trapezoidal and the Simpsons rules looks like* $\int_a^b f(x)dx \approx \sum_k w_k f(x_k)$ *and we knew* x_k *and found* w_k*. In this method we find both* x_k *and* w_k *so that the integral and the sum are equal for a given* n *degree polynomial* $p(x)$ *, by forcing both sides equal for each power of* x^j *for* $j = 0, 1, 2, , n$*. What is the degree of the polynomial we need to use if we want 3 points? Find them for* $[a, b] = [-1, 1]$ *and use it to approximate* $\int_{-1}^{1} e^{-\frac{x^2}{2}} dx$*.*

Quiz 14. *Fit a Least Square Parabola to the data set* $\{(2, 1), (3, 2), (4, 3), (6, 4)\}$ *One method to find the maximum of a multivariate function* $E(a, b, c)$ *is called the Steepest Descent Method. Here we start at a given point* (a_0, b_0, c_0) *and select the direction of the maximum slope at* (a_0, b_0, c_0)*. Then we follow that maximum slope direction till we get the maximum along that direction as a one variable function, say at* (a_1, b_1, c_1) *and we repeat the process. Show that the maximum directions at* (a_0, b_0, c_0) *and* (a_1, b_1, c_1) *are perpendicular. The function to minimize for the given data set is* $E(a, b, c) = 30-428a+1649a^2-88b+630ab+65b^2-20c+130ac+30bc+4c^2$ *Write the first two steps of the Steepest Descend Method starting from* $(0, 0, 0)$*.*

Quiz 15. *(MID Q2) An iterative method of finding solutions to a non-linear equation* $f(x) = 0$ *is said to have a convergence of order* p *iff* $|x_{k+1} - z| \leq r|x_k - z|^p$ *where* x_k *is the* k*th iteration,* z *is the solution and* r *is a constant. Show that* $p = 1$ *for the iterative (fixed point) method and* $p = 2$ *for the Newtons method.*

Quiz 16. *(MID Q4) Find the value of* x *for which* $\int_0^x \frac{1}{\sqrt{pi}} e^{-\frac{t^2}{2}} dt = 0.2$ *using the Newtons method. Note that the above integral cannot be evaluated in closed form but your CASIO calculator can integrate,* $\int_a^b f(x)dx$ *is input as* $\int(f(x), a, b)$*.*

Quiz 17. *(MID Q6) Let* $T_n(x) = \sum_{k=1}^{n} \frac{(-x)^k}{k!}$ *be the* n *th degree Taylor polynomial of* e^{-x} *at* $x = 0$ *and* $\lim_{x\to\infty} T_n(x) = e^{-x}$*. Solve* $x = T_2(x)$ *and find an approximate solution to* $x = e^{-x}$*. Also find a* n *for which the difference in the solutions to* $x = T_n(x)$ *and* $x = e^{-x}$ *is less than 0.001. Assume that one real solution to* $x = T_n(x)$ *remain in* $[0.5, 0.6]$ *for all* $n \geq 2$*.*