Q1. An iterative method of finding solutions to a non-linear equation $f(x) = 0$ is said to have a convergence of order p iff $|x_{k+1} - z| \leq r|x_k - z|^p$ where x_k is the k th iteration, z is the solution and r is a constant. Show that $p = 1$ for the iterative (fixed point) method.

Q2. With the same information as in Q1, show that $p = 2$ for the Newton's method.

Q3. Suppose we want to find the value of x for which $\int_a^x f(t)dt = c$ where f is a given continuous function and a and c are constants. Write the iterative formula that can be used to find x using the Newton's method.

Q4. Find the value of x for which $\int_0^x \frac{1}{\sqrt{\pi}} e^{-\frac{t^2}{2}} dt = 0.2$ using the Newton's method. Note that the above integral cannot be evaluated in closed form but your CASIO calculator can integrate, $\int_a^b f(x)dx$ is input as $\int(f(x), a, b)$.

Q5. Using the iterative (fixed point) method on the interval $[a, b] = [0.5, 0.6]$ with $x_0 = 0.5$, find a number of iterations sufficient to find the real solution of $x = e^{-x}$ accurate to 0.001.
Also show the first 3 iterations and the solution to that accuracy.

Q6. Let $T_n(x) = \sum_{k=0}^{n} \frac{(-x)^k}{k!}$ be n th degree Taylor polynomial of e^{-x} at $x = 0$ and $\lim_{n\to\infty} T_n(x) = e^{-x}$.
Solve $x = T_2(x)$ and find an approximate solution to $x = e^{-x}$.
Also find a n for which the difference in the solutions to $x = T_n(x)$ and $x = e^{-x}$ is less than 0.001.
Assume that one real solution to $x = T_n(x)$ remain in $[0.5,1]$ for all $n \geq 2$.