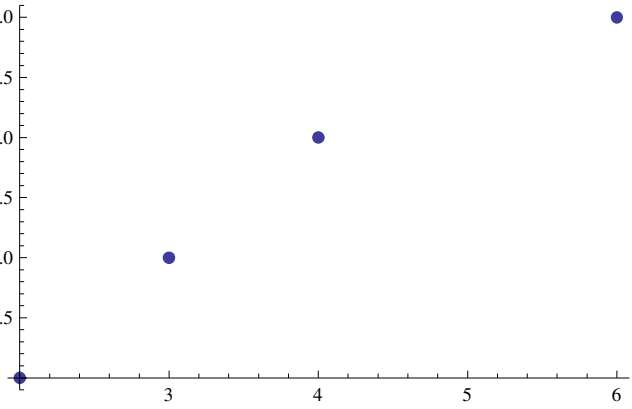
In[1]:=
```
A = {{2, 1}, {3, 2}, {4, 3}, {6, 4}}
```

Out[1]=
```
{{2, 1}, {3, 2}, {4, 3}, {6, 4}}
```

In[2]:=
```
a = ListPlot[A, PlotStyle → PointSize[0.02]]
```

Out[2]=



In[3]:=
```
InterpolatingPolynomial[A, x]
```

Out[3]=
$$1 + \left(1 + \frac{1}{24}\,(4 - x)\,(-3 + x)\right)\,(-2 + x)$$

In[4]:=
```
Expand[%]
```

Out[4]=
$$-\frac{x}{12} + \frac{3\,x^2}{8} - \frac{x^3}{24}$$

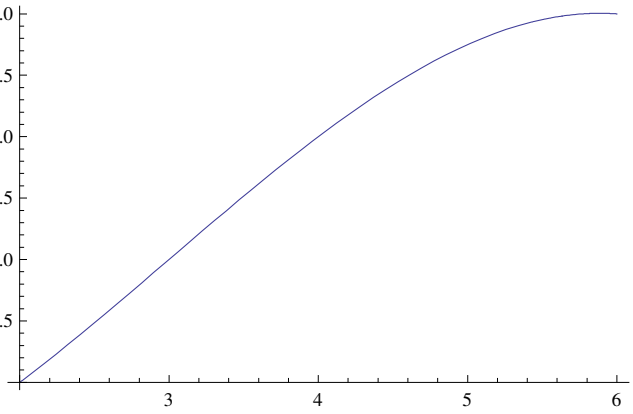
In[5]:=
```
p[x_] := -x/12 + 3 x^2/8 - x^3/24
```

In[6]:=
```
p[{2, 3, 4, 6}]
```

Out[6]=
```
{1, 2, 3, 4}
```

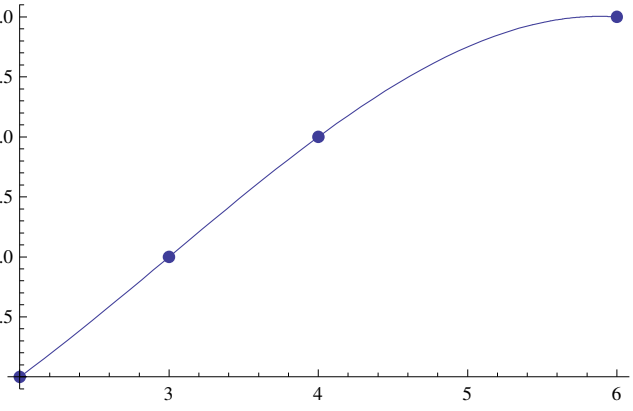
In[7]:=
```
b = Plot[p[x], {x, 2, 6}]
```

Out[7]=



In[8]:=
```
Show[a, b, PlotRange → All]
```

Out[8]=



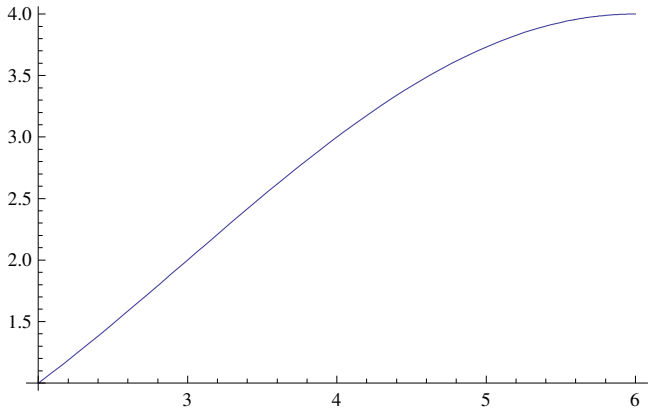
In[9]:=
```
f[x_] := 4 Sin[Pi x / 12]^2
```

In[10]:=
```
f[{2, 3, 4, 6}]
```

Out[10]=
```
{1, 2, 3, 4}
```

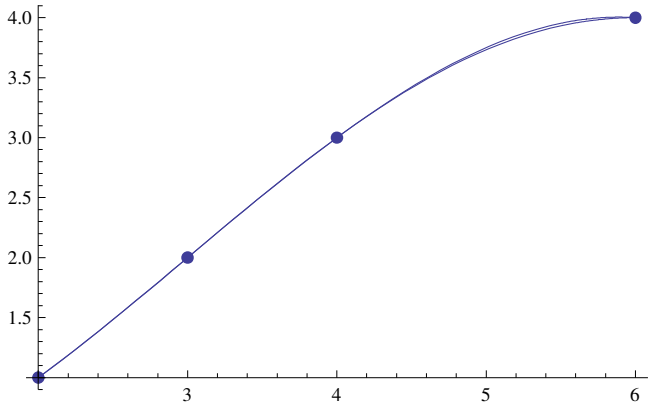
In[11]:=
```
c = Plot[f[x], {x, 2, 6}]
```

Out[11]=



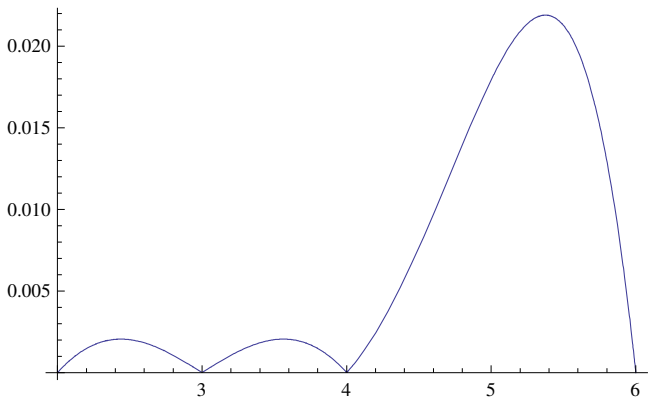
In[12]:=
```
Show[a, b, c, PlotRange → All]
```

Out[12]=



In[13]:=
```
Plot[Abs[f[x] - p[x]], {x, 2, 6}]
```

Out[13]=



In[14]:=
```
f''''[x]
```

Out[14]=
$$-\frac{1}{648}\pi^4 \text{Cos}\left[\frac{\pi x}{12}\right]^2 + \frac{1}{648}\pi^4 \text{Sin}\left[\frac{\pi x}{12}\right]^2$$

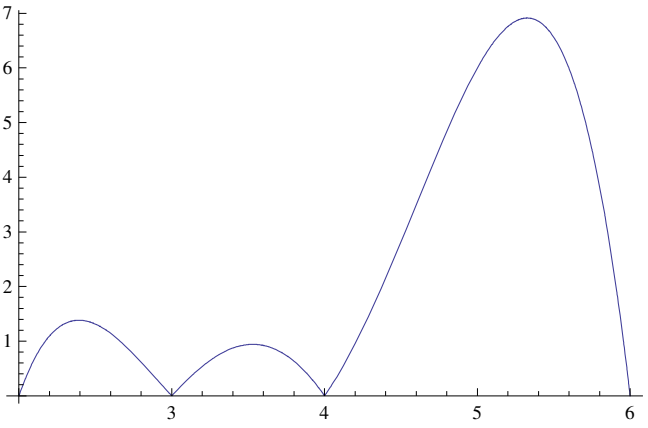
In[15]:=
```
TrigReduce[%]
```

Out[15]=
$$-\frac{1}{648}\pi^4 \text{Cos}\left[\frac{\pi x}{6}\right]$$

In[16]:=
```
W[x_] := (x - 2) (x - 3) (x - 4) (x - 6)
```

In[17]:=
```
Plot[Abs[W[x]], {x, 2, 6}]
```

Out[17]=



In[18]:=
```
W'[x]
```

Out[18]=
$$(-6+x)\,(-4+x)\,(-3+x)+(-6+x)\,(-4+x)\,(-2+x)+(-6+x)\,(-3+x)\,(-2+x)+(-4+x)\,(-3+x)\,(-2+x)$$

In[19]:=
```
Solve[% == 0, x]
```

Out[19]=
$$\left\{\left\{x\to\frac{15}{4}+\frac{7\,5^{2/3}}{4\left(3\left(27+8\,i\sqrt{69}\right)\right)^{1/3}}+\frac{\left(5\left(27+8\,i\sqrt{69}\right)\right)^{1/3}}{4\,3^{2/3}}\right\},\right.$$
$$\left\{x\to\frac{15}{4}-\frac{7\,5^{2/3}\left(1+i\sqrt{3}\right)}{8\left(3\left(27+8\,i\sqrt{69}\right)\right)^{1/3}}-\frac{\left(1-i\sqrt{3}\right)\left(5\left(27+8\,i\sqrt{69}\right)\right)^{1/3}}{8\,3^{2/3}}\right\},$$
$$\left.\left\{x\to\frac{15}{4}-\frac{7\,5^{2/3}\left(1-i\sqrt{3}\right)}{8\left(3\left(27+8\,i\sqrt{69}\right)\right)^{1/3}}-\frac{\left(1+i\sqrt{3}\right)\left(5\left(27+8\,i\sqrt{69}\right)\right)^{1/3}}{8\,3^{2/3}}\right\}\right\}$$

In[20]:=
```
N[%, 20]
```

Out[20]=
$$\{\{x\to 5.3263454633578327541+0.\times 10^{-20}\,i\},$$
$$\{x\to 2.3927479811269487269+0.\times 10^{-20}\,i\},\ \{x\to 3.5309065555152185190+0.\times 10^{-20}\,i\}\}$$

In[21]:=
```
W[x] /. %
```

Out[21]=
$$\{-6.9140967887662470\,36+0.\times 10^{-19}\,i,$$
$$-1.3827491294425262+0.\times 10^{-19}\,i,\ 0.9413771682087732\,79+0.\times 10^{-19}\,i\}$$

In[22]:=
```
W[{2, 6}]
```

Out[22]=
$$\{0,\ 0\}$$

In[23]:=
```
Maximize[{Abs[W[x]], 2 ≤ x ≤ 6}, x]
```

Out[23]=
$$\{-(-6+\text{Root}[-180+160\,\#1-45\,\#1^2+4\,\#1^3\ \&,\ 3])\,(-4+\text{Root}[-180+160\,\#1-45\,\#1^2+4\,\#1^3\ \&,\ 3])$$
$$(-3+\text{Root}[-180+160\,\#1-45\,\#1^2+4\,\#1^3\ \&,\ 3])\,(-2+\text{Root}[-180+160\,\#1-45\,\#1^2+4\,\#1^3\ \&,\ 3]),$$
$$\{x\to\text{Root}[-180+160\,\#1-45\,\#1^2+4\,\#1^3\ \&,\ 3]\}\}$$

In[24]:=
```
N[%, 20]
```

Out[24]=
$$\{6.9140967887662470361,\ \{x\to 5.3263454633578327541\}\}$$

In[25]:=
```
1/648 π^4 (6.914096788766247036109449544209756341 76`20.) / 4!
```

Out[25]=
$$0.043306062468803708679$$

In[26]:=
```
FindMaximum[{Abs[f[x] - p[x]], 2 ≤ x ≤ 6}, {x, 5}]
```

Out[26]=
$$\{0.0219112,\ \{x\to 5.37469\}\}$$