

In[1]:= **Table** $\left[\int_{-1}^1 \mathbf{x}^n dx = \text{Sum}[w[k] \mathbf{x}[k]^n, \{k, 0, 2\}], \{n, 0, 5\} \right]$

Out[1]= $\left\{ \begin{aligned} 2 &= w[0] + w[1] + w[2], 0 = w[0] x[0] + w[1] x[1] + w[2] x[2], \\ \frac{2}{3} &= w[0] x[0]^2 + w[1] x[1]^2 + w[2] x[2]^2, 0 = w[0] x[0]^3 + w[1] x[1]^3 + w[2] x[2]^3, \\ \frac{2}{5} &= w[0] x[0]^4 + w[1] x[1]^4 + w[2] x[2]^4, 0 = w[0] x[0]^5 + w[1] x[1]^5 + w[2] x[2]^5 \end{aligned} \right\}$

In[2]:= **Solve**[% , {w[0], w[1], w[2], x[0], x[1], x[2]}]

Out[2]= $\left\{ \left\{ w[0] \rightarrow \frac{5}{9}, w[1] \rightarrow \frac{5}{9}, w[2] \rightarrow \frac{8}{9}, x[2] \rightarrow 0, x[1] \rightarrow -\sqrt{\frac{3}{5}}, x[0] \rightarrow \sqrt{\frac{3}{5}} \right\}, \right.$
 $\left. \left\{ w[0] \rightarrow \frac{5}{9}, w[1] \rightarrow \frac{5}{9}, w[2] \rightarrow \frac{8}{9}, x[2] \rightarrow 0, x[1] \rightarrow \sqrt{\frac{3}{5}}, x[0] \rightarrow -\sqrt{\frac{3}{5}} \right\}, \right.$
 $\left\{ w[0] \rightarrow \frac{5}{9}, w[1] \rightarrow \frac{8}{9}, w[2] \rightarrow \frac{5}{9}, x[2] \rightarrow -\sqrt{\frac{3}{5}}, x[1] \rightarrow 0, x[0] \rightarrow \sqrt{\frac{3}{5}} \right\}, \right.$
 $\left\{ w[0] \rightarrow \frac{5}{9}, w[1] \rightarrow \frac{8}{9}, w[2] \rightarrow \frac{5}{9}, x[2] \rightarrow \sqrt{\frac{3}{5}}, x[1] \rightarrow 0, x[0] \rightarrow -\sqrt{\frac{3}{5}} \right\}, \right.$
 $\left\{ w[0] \rightarrow \frac{8}{9}, w[1] \rightarrow \frac{5}{9}, w[2] \rightarrow \frac{5}{9}, x[2] \rightarrow -\sqrt{\frac{3}{5}}, x[1] \rightarrow \sqrt{\frac{3}{5}}, x[0] \rightarrow 0 \right\}, \right.$
 $\left. \left\{ w[0] \rightarrow \frac{8}{9}, w[1] \rightarrow \frac{5}{9}, w[2] \rightarrow \frac{5}{9}, x[2] \rightarrow \sqrt{\frac{3}{5}}, x[1] \rightarrow -\sqrt{\frac{3}{5}}, x[0] \rightarrow 0 \right\} \right\}$

In[3]:= **%**[4]

Out[3]= $\left\{ w[0] \rightarrow \frac{5}{9}, w[1] \rightarrow \frac{8}{9}, w[2] \rightarrow \frac{5}{9}, x[2] \rightarrow \sqrt{\frac{3}{5}}, x[1] \rightarrow 0, x[0] \rightarrow -\sqrt{\frac{3}{5}} \right\}$

In[4]:= **f**[x_] := **Sin**[x^2]

In[5]:= **g**[t_] := **Sin**[((t + 1) / 2)^2] / 2

In[6]:= $\frac{5}{9} g\left[-\sqrt{\frac{3}{5}}\right] + \frac{8}{9} g[0] + \frac{5}{9} g\left[\sqrt{\frac{3}{5}}\right]$

Out[6]= $\frac{4}{9} \text{Sin}\left[\frac{1}{4}\right] + \frac{5}{18} \text{Sin}\left[\frac{1}{4} \left(1 - \sqrt{\frac{3}{5}}\right)^2\right] + \frac{5}{18} \text{Sin}\left[\frac{1}{4} \left(1 + \sqrt{\frac{3}{5}}\right)^2\right]$

In[7]:= **N**[% , 20]

Out[7]= 0.31027688512104177889

In[8]:= **NIntegrate**[f[x], {x, 0, 1}, WorkingPrecision -> 20]

Out[8]= 0.31026830172338110179

In[9]:= **%** - **%%**

Out[9]= $-8.58339766067710 \times 10^{-6}$