

```
In[1]:= f[x_] := x - E^(-x)
```

```
In[2]:= f[0.3]
```

```
Out[2]= -0.440818
```

```
In[3]:= f[1]
```

```
Out[3]= 1 -  $\frac{1}{e}$ 
```

```
In[4]:= N[%]
```

```
Out[4]= 0.632121
```

```
In[5]:= g[x_] := E^(-x)
```

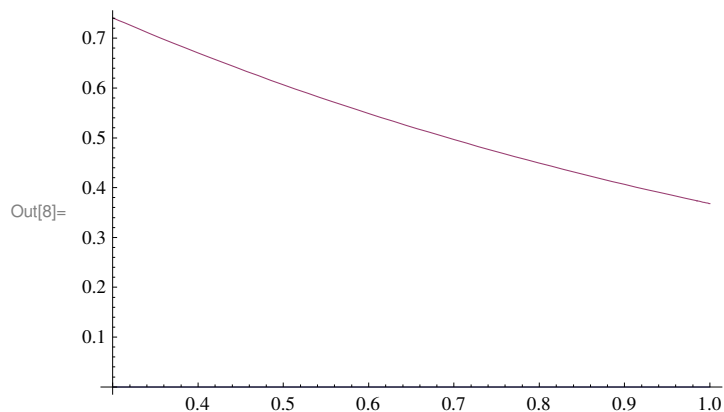
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In[6]:= N[Abs[g'[1]]]
```

```
Out[6]= 0.367879
```

```
In[7]:= N[Abs[g'[0.3]]]
```

```
Out[7]= 0.740818
```

```
In[8]:= Plot[{0, Abs[g'[x]]}, {x, 0.3, 1}]
```



```
In[9]:= L = Abs[g'[0.3]]
```

```
Out[9]= 0.740818
```

```
In[12]:= N[Log[10^(-6) (1 - L) / Abs[0.5 - g[0.5]]] / Log[L]]
```

```
Out[12]= 43.088
```

```
In[13]:= x = 1 / 2; For[k = 0, k ≤ 44, k++, {Print[N[{k, x, Abs[x - g[x]]}, 20]], x = g[x]}]
```

```
{0, 0.50000000000000000000, 0.10653065971263342360}
{1.00000000000000000000, 0.60653065971263342360, 0.061291447820028368184}
{2.00000000000000000000, 0.54523921189260505542, 0.034463882985463154443}
{3.00000000000000000000, 0.57970309487806820986, 0.019638466939166345847}
{4.00000000000000000000, 0.56006462793890186402, 0.011107521038313291350}
{5.00000000000000000000, 0.57117214897721515537, 0.0063092019968917140133}
{6.00000000000000000000, 0.56486294698032344135, 0.0035751005897428142459}
{7.00000000000000000000, 0.56843804757006625560, 0.0020285948231454421898}
{8.00000000000000000000, 0.56640945274692081341, 0.0011501815153216138062}
{9.00000000000000000000, 0.56755963426224242722, 0.00065242132677103481030}
{10.00000000000000000000, 0.56690721293547139240, 0.00036998303530710160875}
{11.00000000000000000000, 0.56727719597077849401, 0.00020984411705033422211}
{12.00000000000000000000, 0.56706735185372815979, 0.00011900823390999387117}
{13.00000000000000000000, 0.56718636008763815366, 0.000067495830652404810286}
{14.00000000000000000000, 0.56711886425698574885, 0.000038279450658800794741}
```

```
{15.000000000000000000, 0.56715714370764454965, 0.000021710048371329448542}
{16.000000000000000000, 0.56713543365927322020, 0.000012312671351699430873}
{17.000000000000000000, 0.56714774633062491963, 6.9830608182713188866 × 10-6}
{18.000000000000000000, 0.56714076326980664831, 3.9603922702478266151 × 10-6}
{19.000000000000000000, 0.56714472366207689614, 2.2461111319596106240 × 10-6}
{20.000000000000000000, 0.56714247755094493653, 1.2738664628566168581 × 10-6}
{21.000000000000000000, 0.56714375141740779314, 7.2246494438729632933 × 10-7}
{22.000000000000000000, 0.56714302895246340585, 4.0974110488355872071 × 10-7}
{23.000000000000000000, 0.56714343869356828941, 2.3238193158954556634 × 10-7}
{24.000000000000000000, 0.56714320631163669986, 1.3179384908384030462 × 10-7}
{25.000000000000000000, 0.56714333810548578370, 7.4745998585648272877 × 10-8}
{26.000000000000000000, 0.56714326335948719805, 4.2391691145232799503 × 10-8}
{27.000000000000000000, 0.56714330575117834329, 2.4042163342896300180 × 10-8}
{28.000000000000000000, 0.56714328170901500039, 1.3635351581586064503 × 10-8}
{29.000000000000000000, 0.56714329534436658197, 7.7331981764373032538 × 10-9}
{30.000000000000000000, 0.56714328761116840554, 4.3858314544915963971 × 10-9}
{31.000000000000000000, 0.56714329199699986003, 2.4873948837897066942 × 10-9}
{32.000000000000000000, 0.56714328950960497624, 1.4107093184563514195 × 10-9}
{33.000000000000000000, 0.56714329092031429470, 8.0007432483695266398 × 10-10}
{34.000000000000000000, 0.56714329012023996986, 4.5375678511027853684 × 10-10}
{35.000000000000000000, 0.56714329057399675497, 2.5734511616933525811 × 10-10}
{36.000000000000000000, 0.56714329031665163880, 1.4595155594997770823 × 10-10}
{37.000000000000000000, 0.56714329046260319475, 8.2775445683566486386 × 10-11}
{38.000000000000000000, 0.56714329037982774907, 4.6945538629577576176 × 10-11}
{39.000000000000000000, 0.56714329042677328770, 2.6624847248610860829 × 10-11}
{40.000000000000000000, 0.56714329040014844045, 1.5100103475179521618 × 10-11}
{41.000000000000000000, 0.56714329041524854392, 8.5639223704593851553 × 10-12}
{42.000000000000000000, 0.56714329040668462155, 4.8569711119905473984 × 10-12}
{43.000000000000000000, 0.56714329041154159266, 2.7545985778814336245 × 10-12}
{44.000000000000000000, 0.56714329040878699409, 1.5622521012171932570 × 10-12}
```

In[14]:= **N[x, 20]**

Out[14]= 0.56714329041034924619

In[15]:= **FindRoot[f[y] == 0, {y, 1/2}, WorkingPrecision → 20]**

Out[15]= {y → 0.56714329040978387300}

In[16]:= **%[[1]][[2]] - N[x, 20]**

Out[16]= -5.653732 × 10⁻¹³