

In[1]:= $p[k_, x_] := a[k] x^3 + b[k] x^2 + c[k] x + d[k]$

In[2]:= $D[p[k, x], x]$

In[3]:= $p'[k_, x_] := 3 x^2 a[k] + 2 x b[k] + c[k]$

In[4]:= $D[p[1, x], \{x, 2\}]$

In[5]:= $p''[k_, x_] := 6 x a[k] + 2 b[k]$

In[6]:= $A = \{\{2, 1\}, \{3, 2\}, \{4, 3\}, \{6, 4\}\}$

Out[6]= $\{\{2, 1\}, \{3, 2\}, \{4, 3\}, \{6, 4\}\}$

In[7]:= $B = \{p[1, 2] = 1, p[1, 3] = 2, p[2, 3] = 2, p[2, 4] = 3, p[3, 4] = 3, p[3, 6] = 4,$
 $p'[1, 3] = p'[2, 3], p'[2, 4] = p'[3, 4],$
 $p''[1, 3] = p''[2, 3], p''[2, 4] = p''[3, 4],$
 $p''[1, 2] = 0, p''[3, 6] = 0\}$

Out[7]= $\{8 a[1] + 4 b[1] + 2 c[1] + d[1] = 1, 27 a[1] + 9 b[1] + 3 c[1] + d[1] = 2, 27 a[2] + 9 b[2] + 3 c[2] + d[2] = 2,$
 $64 a[2] + 16 b[2] + 4 c[2] + d[2] = 3, 64 a[3] + 16 b[3] + 4 c[3] + d[3] = 3,$
 $216 a[3] + 36 b[3] + 6 c[3] + d[3] = 4, 27 a[1] + 6 b[1] + c[1] = 27 a[2] + 6 b[2] + c[2],$
 $48 a[2] + 8 b[2] + c[2] = 48 a[3] + 8 b[3] + c[3], 18 a[1] + 2 b[1] = 18 a[2] + 2 b[2],$
 $24 a[2] + 2 b[2] = 24 a[3] + 2 b[3], 12 a[1] + 2 b[1] = 0, 36 a[3] + 2 b[3] = 0\}$

In[8]:= $B // \text{TableForm}$

Out[8]/TableForm=

$$\begin{aligned} 8 a[1] + 4 b[1] + 2 c[1] + d[1] &= 1 \\ 27 a[1] + 9 b[1] + 3 c[1] + d[1] &= 2 \\ 27 a[2] + 9 b[2] + 3 c[2] + d[2] &= 2 \\ 64 a[2] + 16 b[2] + 4 c[2] + d[2] &= 3 \\ 64 a[3] + 16 b[3] + 4 c[3] + d[3] &= 3 \\ 216 a[3] + 36 b[3] + 6 c[3] + d[3] &= 4 \\ 27 a[1] + 6 b[1] + c[1] &= 27 a[2] + 6 b[2] + c[2] \\ 48 a[2] + 8 b[2] + c[2] &= 48 a[3] + 8 b[3] + c[3] \\ 18 a[1] + 2 b[1] &= 18 a[2] + 2 b[2] \\ 24 a[2] + 2 b[2] &= 24 a[3] + 2 b[3] \\ 12 a[1] + 2 b[1] &= 0 \\ 36 a[3] + 2 b[3] &= 0 \end{aligned}$$

In[9]:= $\text{Flatten}[\text{Table}\{a[k], b[k], c[k], d[k]\}, \{k, 1, 3\}]$

In[10]:= $T = \{a[1], b[1], c[1], d[1], a[2], b[2], c[2], d[2], a[3], b[3], c[3], d[3]\}$

Out[10]= $\{a[1], b[1], c[1], d[1], a[2], b[2], c[2], d[2], a[3], b[3], c[3], d[3]\}$

In[11]:= $\text{Solve}[B, T]$

In[12]:= $S = \left\{ a[1] \rightarrow \frac{1}{46}, b[1] \rightarrow -\frac{3}{23}, c[1] \rightarrow \frac{57}{46}, d[1] \rightarrow -\frac{26}{23}, a[2] \rightarrow -\frac{5}{46}, b[2] \rightarrow \frac{24}{23}, \right.$
 $\left. c[2] \rightarrow -\frac{105}{46}, d[2] \rightarrow \frac{55}{23}, a[3] \rightarrow \frac{1}{23}, b[3] \rightarrow -\frac{18}{23}, c[3] \rightarrow \frac{231}{46}, d[3] \rightarrow -\frac{169}{23} \right\};$

In[13]:= $p[1, x] /. S$

In[14]:= $p1[x_] := -\frac{26}{23} + \frac{57 x}{46} - \frac{3 x^2}{23} + \frac{x^3}{46}$

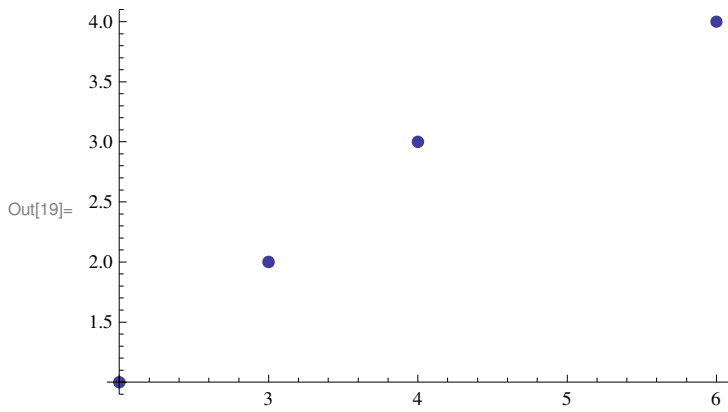
In[15]:= $p[2, x] /. S$

In[16]:= $p2[x_] := \frac{55}{23} - \frac{105 x}{46} + \frac{24 x^2}{23} - \frac{5 x^3}{46}$

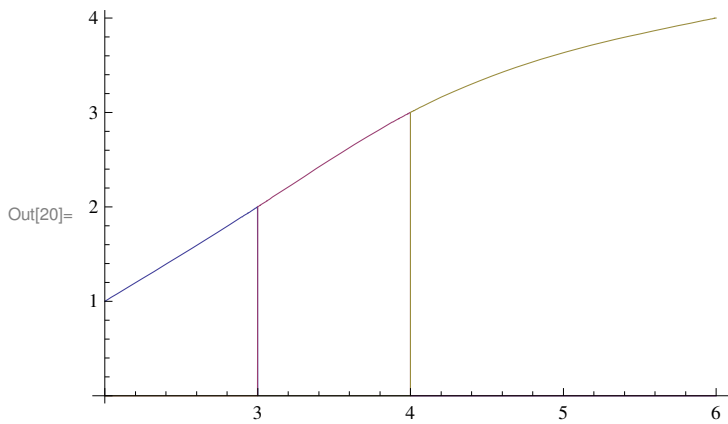
In[17]:= $p[3, x] /. S$

In[18]:= $p3[x_] := -\frac{169}{23} + \frac{231 x}{46} - \frac{18 x^2}{23} + \frac{x^3}{23}$

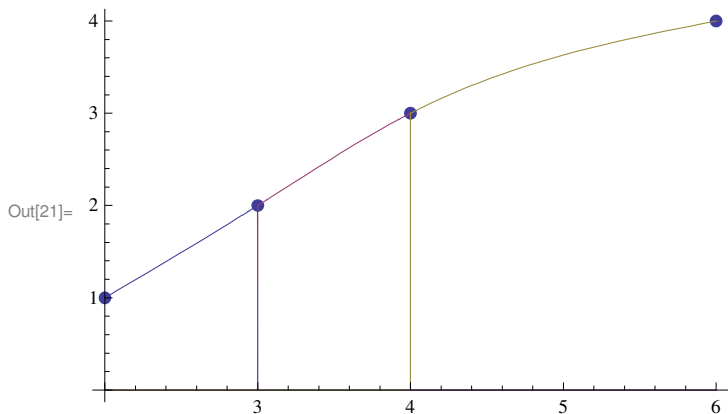
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In[19]:= q1 = ListPlot[A, PlotStyle -> PointSize[0.02]]
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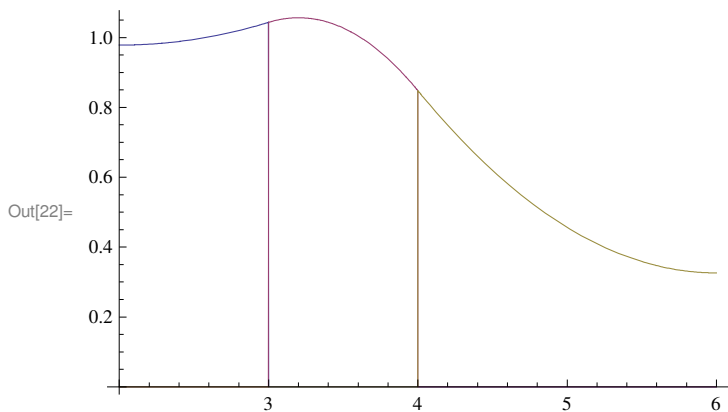
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In[20]:= q2 = Plot[{p1[x] UnitBox[x - 2.5], p2[x] UnitBox[x - 3.5], p3[x] UnitBox[(x - 5) / 2]}, {x, 2, 6}]
```



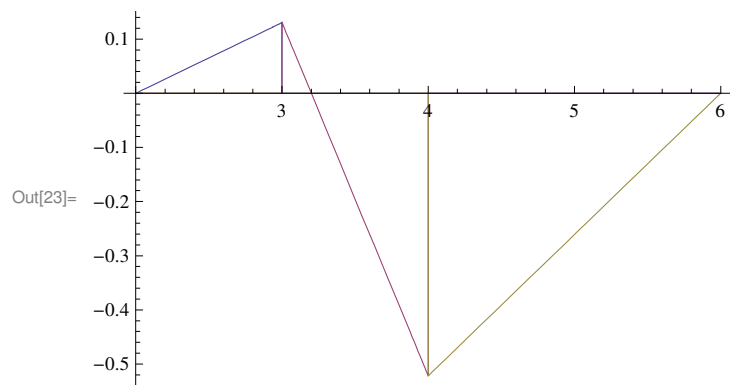
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In[21]:= Show[q1, q2]
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In[22]:= Plot[{p1'[x] UnitBox[x - 2.5], p2'[x] UnitBox[x - 3.5], p3'[x] UnitBox[(x - 5) / 2]}, {x, 2, 6}]
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In[23]:= Plot[{p1''[x] UnitBox[x - 2.5], p2''[x] UnitBox[x - 3.5], p3''[x] UnitBox[(x - 5) / 2]},  
{x, 2, 6}, PlotRange -> All]
```



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In[24]:= Plot[{p1'''[x] UnitBox[x - 2.5], p2'''[x] UnitBox[x - 3.5], p3'''[x] UnitBox[(x - 5) / 2]},  
{x, 2, 6}, PlotRange -> All]
```

