

In[1]:= $f[x_] := x - E^{-x}$

In[2]:= $f[0]$

Out[2]= -1

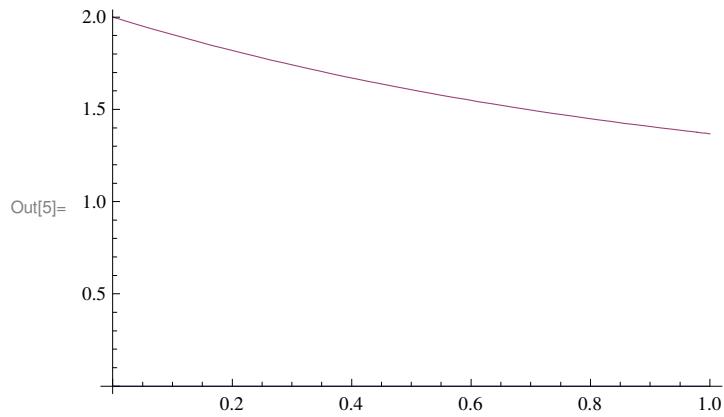
In[3]:= $f[1]$

Out[3]= $1 - \frac{1}{e}$

In[4]:= $f'[x]$

Out[4]= $1 + e^{-x}$

In[5]:= $\text{Plot}[\{0, f'[x]\}, \{x, 0, 1\}]$



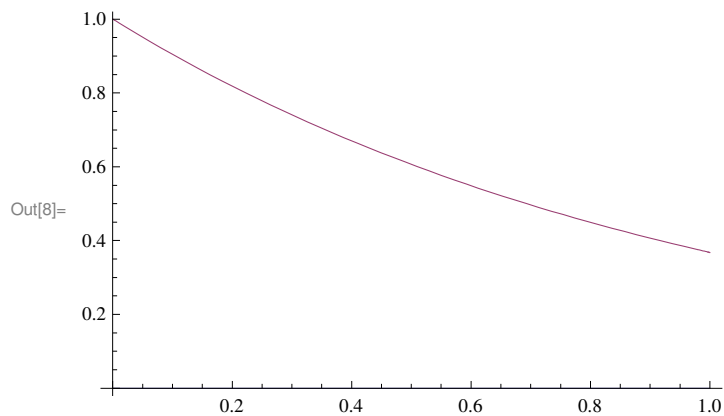
In[6]:= $b = 1 / f'[1]$

Out[6]= $\frac{1}{1 + \frac{1}{e}}$

In[7]:= $f''[x]$

Out[7]= $-e^{-x}$

In[8]:= $\text{Plot}[\{0, \text{Abs}[f''[x]]\}, \{x, 0, 1\}]$



In[10]:= $g = \text{Abs}[f''[0]]$

Out[10]= 1

In[11]:= $x = 1 / 2$

Out[11]= $\frac{1}{2}$

In[12]:= $a = \text{Abs}[f[x] / f'[x]]$

Out[12]= $\frac{-\frac{1}{2} + \frac{1}{\sqrt{e}}}{1 + \frac{1}{\sqrt{e}}}$

In[13]:= **q = a b g**

$$\text{Out[13]} = \frac{-\frac{1}{2} + \frac{1}{\sqrt{e}}}{\left(1 + \frac{1}{e}\right) \left(1 + \frac{1}{\sqrt{e}}\right)}$$

In[14]:= **N[q]**

Out[14]= 0.0484772

In[15]:= **% < 1 / 2**

Out[15]= True

In[16]:= **{x - 2 a, x + 2 a}**

$$\text{Out[16]} = \left\{ \frac{1}{2} - \frac{2 \left(-\frac{1}{2} + \frac{1}{\sqrt{e}}\right)}{1 + \frac{1}{\sqrt{e}}}, \frac{1}{2} + \frac{2 \left(-\frac{1}{2} + \frac{1}{\sqrt{e}}\right)}{1 + \frac{1}{\sqrt{e}}} \right\}$$

In[17]:= **N[%]**

Out[17]= {0.367378, 0.632622}

In[18]:= **Table[N[{k, -2 a q^(2^k - 1) + 10^(-6)}], {k, 1, 10}]**

Out[18]= {{1., -0.00642815}, {2., -0.0000141088}, {3., 9.99917 × 10⁻⁷}, {4., 1. × 10⁻⁶},
{5., 1. × 10⁻⁶}, {6., 1. × 10⁻⁶}, {7., 1. × 10⁻⁶}, {8., 1. × 10⁻⁶}, {9., 1. × 10⁻⁶}, {10., 1. × 10⁻⁶}}

In[19]:= **Log[Log[10^(-6) / (2 a)] / Log[q] + 1] / Log[2]**

$$\text{Out[19]} = \frac{\text{Log}\left[1 + \frac{\text{Log}\left[\frac{1 + \frac{1}{\sqrt{e}}}{2000000 \left(\frac{1}{2} + \frac{1}{\sqrt{e}}\right)}\right]}{\text{Log}\left[\frac{1 + \frac{1}{\sqrt{e}}}{\left(1 + \frac{1}{e}\right) \left(1 + \frac{1}{\sqrt{e}}\right)}\right]}\right]}{\text{Log}[2]}$$

In[20]:= **N[%]**

Out[20]= 2.29193

In[21]:= **x**

$$\text{Out[21]} = \frac{1}{2}$$

In[22]:= **For[k = 1, k ≤ 3, k++, x = x - f[x] / f'[x]]**

In[23]:= **A = N[x, 30]**

Out[23]= 0.567143290409781028699576649415

In[24]:= **Clear[x]**

In[25]:= **B = FindRoot[f[x] == 0, {x, 1}, WorkingPrecision → 30]**

Out[25]= {x → 0.567143290409783872999968662210}

In[26]:= **A - B[[1]][[2]]**

Out[26]= -2.84430039201280 × 10⁻¹⁵