Quiz 1.
1. Suppose we want to solve $\tan^{-1} x = 0$ *by the Newton's method. Find the value* z *such that the Newton's method is converging for* $0 < x_0 < z$*, diverging for* $x_0 > z$ *and enters into a cycle for* $x_0 = z$*.*
2. Your CASIO calculator can integrate, $\int_a^b f(x)dx$ *is evaluated as* $\int(f(x), a, b)$*. Find the* z *value for which* $P(x < z) = 0.8$ *when* $X \sim N(0, 1)$*, ie when* X *is Normally distributed with mean 0 and standard deviation 1 which is having a PDF* $p(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$*.*
3. Find the height of the circular sector with arc length $2x$ *and chord length* x

Quiz 2. *For the data set* $\{(x_k, y_k)\}, k = 0, 1, , n$ *a natural cubic spline is a twice differentiable piecewise cubic polynomial* $p(x)$ *which satisfies* $p(x_k) = y_k$ *with* $p''(x_0) = p''(x_n) = 0$*. Let* $p(x) = \sum_{k=0}^{n-1} p_k(x)$ *where* $p_k(x)$ *is the part of* $p(x)$ *on* $[x_k, x_{k+1}]$ *which is 0 elsewhere. Assume that* $p_k(x) = a_k(x-x_k)^3 + b_k(x-x_k)^2 + c_k(x-x_k) + d_k$ *and that* $x_{k+1} - x_k = h$ *is a constant. With* $s_k = p''(x_k)$ *derive the formula* $s_{k+2} + 4s_{k+1} + s_k = \frac{6}{h^2} h^2 \left(y_{k+2} - 2y_{k+1} + y_k\right), k = 0, 1, , n-2$*. Also write the system of equations in matrix form that must be solved find* s_k *for* $k = 1, , n-1$*. Get the value of USD w.r.t. LKR for the 1st day of every month this year and predict the value for the 1st February next year using natural cubic splines (just show the data set, the calculated* s_k *values and the final answer).*

Quiz 3. *One method of doing numerical integration is Gaussian Quadrature. Note that both the Trapezoidal and the Simpsons rules looks like* $\int_a^b f(x)dx \approx \sum_k w_k f(x_k)$ *and we knew* x_k *and found* w_k*. In this method we find both* x_k *and* w_k *so that the integral and the sum are equal for a given* n *degree polynomial* $p(x)$ *, by forcing both sides equal for each power of* x^j *for* $j = 0, 1, 2, , n$*. What is the degree of the polynomial we need to use if we want 3 points? Find them for* $[a, b] = [-1, 1]$ *and use it to approximate* $\int_{-1}^{1} e^{-\frac{x^2}{2}} dx$*.*

Quiz 4. *(MID C2) Let* $T_n(x) = \sum_{k=1}^{n} \frac{(-x)^k}{k!}$ *be the* n *th degree Taylor polynomial of* e^{-x} *at* $x = 0$ *and* $\lim_{x\to\infty} T_n(x) = e^{-x}$*. Solve* $x = T_2(x)$ *and find an approximate solution to* $x = e^{-x}$*. Also find a* n *for which the difference in the solutions to* $x = T_n(x)$ *and* $x = e^{-x}$ *is less than 0.001. Assume that one real solution to* $x = T_n(x)$ *remain in* $[0.5, 0.6]$ *for all* $n \geq 2$*.*