In[1]:=
```
f[x_] := x^5 - x - 1
```

In[2]:=
```
f[2]
```

Out[2]=
```
29
```

In[3]:=
```
f[1]
```

Out[3]=
```
-1
```

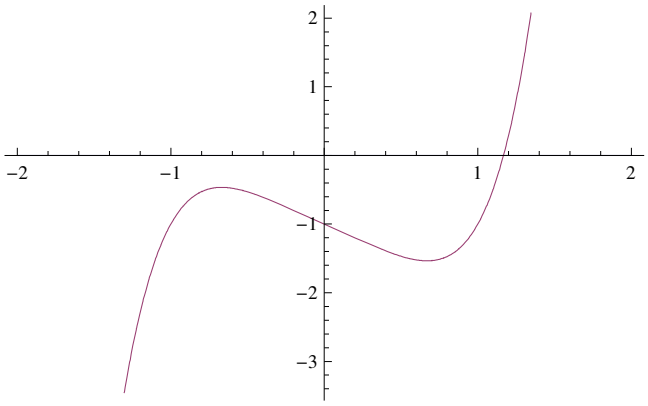
In[4]:=
```
a = 1;
b = 2;
k = 0;
While[
 Abs[b - a] >= 0.0001,
 {x = (a + b) / 2,
  Print[{k, N[{a, b, x}, 15], N[f[x]]}],
  If[
   f[x] f[b] > 0,
   b = x,
   a = x
  ],
  k++
 }
]
```

```
{0, {1.00000000000000, 2.00000000000000, 1.50000000000000}, 5.09375}
{1, {1.00000000000000, 1.50000000000000, 1.25000000000000}, 0.801758}
{2, {1.00000000000000, 1.25000000000000, 1.12500000000000}, -0.322968}
{3, {1.12500000000000, 1.25000000000000, 1.18750000000000}, 0.173892}
{4, {1.12500000000000, 1.18750000000000, 1.15625000000000}, -0.089639}
{5, {1.15625000000000, 1.18750000000000, 1.17187500000000}, 0.0381971}
{6, {1.15625000000000, 1.17187500000000, 1.16406250000000}, -0.0266837}
{7, {1.16406250000000, 1.17187500000000, 1.16796875000000}, 0.00551359}
{8, {1.16406250000000, 1.16796875000000, 1.16601562500000}, -0.0106455}
{9, {1.16601562500000, 1.16796875000000, 1.16699218750000}, -0.00258113}
{10, {1.16699218750000, 1.16796875000000, 1.16748046875000}, 0.00146243}
{11, {1.16699218750000, 1.16748046875000, 1.16723632812500}, -0.000560299}
{12, {1.16723632812500, 1.16748046875000, 1.16735839843750}, 0.00045083}
{13, {1.16723632812500, 1.16735839843750, 1.16729736328125}, -0.0000547937}
```

In[8]:=
```
Clear[x]
```

In[9]:=
```
Plot[{0, f[x]}, {x, -2, 2}]
```

Out[9]=



In[10]:=
```
NSolve[f[x] == 0, x, 15]
```

Out[10]=
```
{{x → -0.764884433600585 - 0.352471546031726 i},
 {x → -0.764884433600585 + 0.352471546031726 i}, {x → 0.181232444469875 - 1.083954101317711 i},
 {x → 0.181232444469875 + 1.083954101317711 i}, {x → 1.16730397826142}}
```

In[1]:=
```
g[x_] := (1 + x) ^ (1 / 5)
```

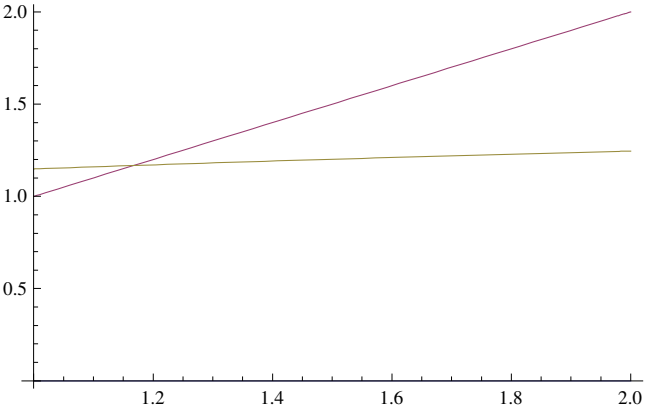
In[2]:=
```
x = 10;
k = 0;
While[k < 20,
 {Print[{k, N[x, 20], N[x - g[x]]}],
  x = N[g[x], 100],
  k++
 }]
```

```
{0, 10.000000000000000000, 8.38461}
{1, 1.6153942662021780015, 0.403381}
{2, 1.2120134336232450842, 0.0399326}
{3, 1.1720808006841303948, 0.00426272}
{4, 1.1678180819102245418, 0.00045873}
{5, 1.1673593519706498210, 0.0000494089}
{6, 1.1673099430260429378, 5.32224 × 10^-6}
{7, 1.1673046207818846788, 5.73309 × 10^-7}
{8, 1.1673040474733626157, 6.17565 × 10^-8}
{9, 1.1673039857168912227, 6.65237 × 10^-9}
{10, 1.1673039790645180953, 7.1659 × 10^-10}
{11, 1.1673039783479281123, 7.71907 × 10^-11}
{12, 1.1673039782707374324, 8.31494 × 10^-12}
{13, 1.1673039782624224947, 8.95681 × 10^-13}
{14, 1.1673039782615268142, 9.64822 × 10^-14}
{15, 1.1673039782614303320, 1.0393 × 10^-14}
{16, 1.1673039782614199389, 1.11953 × 10^-15}
{17, 1.1673039782614188194, 1.20595 × 10^-16}
{18, 1.1673039782614186988, 1.29904 × 10^-17}
{19, 1.1673039782614186858, 1.39932 × 10^-18}
```

In[5]:=
```
Clear[x]
```

In[6]:=
```
Plot[{0, x, g[x]}, {x, 1, 2}]
```

Out[6]=



In[7]:=
```
NSolve[x == g[x], x, 15]
```

Out[7]=
```
{{x → 1.16730397826142}}
```

In[1]:=
```
A = {{2, 1}, {3, 2}, {4, 3}, {6, 4}}
```

Out[1]=
```
{{2, 1}, {3, 2}, {4, 3}, {6, 4}}
```

In[2]:=
```
InterpolatingPolynomial[A, x]
```

Out[2]=
$$1 + \left(1 + \frac{1}{24}\,(4 - x)\,(-3 + x)\right)\,(-2 + x)$$

In[3]:=
```
Expand[%]
```

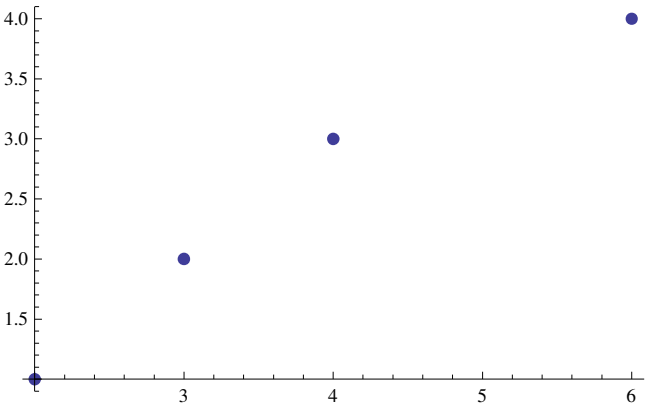
In[4]:=
$$p[x\_] := -\frac{x}{12} + \frac{3\,x^2}{8} - \frac{x^3}{24}$$

In[5]:=
```
p[{2, 3, 4, 6}]
```

Out[5]=
```
{1, 2, 3, 4}
```

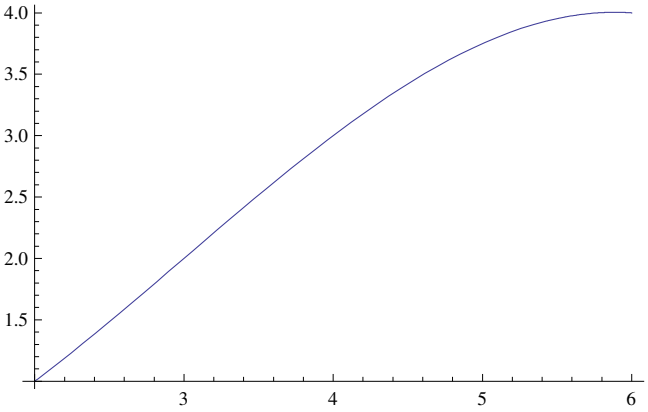
In[6]:=
```
a1 = ListPlot[A, PlotStyle -> PointSize[0.02]]
```

Out[6]=



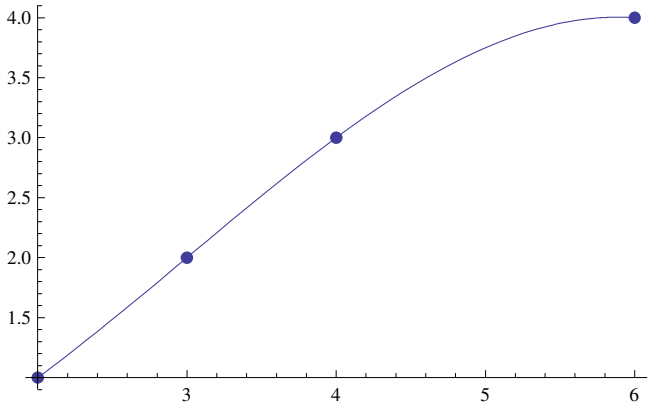
In[7]:=
```
a2 = Plot[p[x], {x, 2, 6}]
```

Out[7]=



In[8]:=
```
Show[a1, a2, PlotRange → All]
```

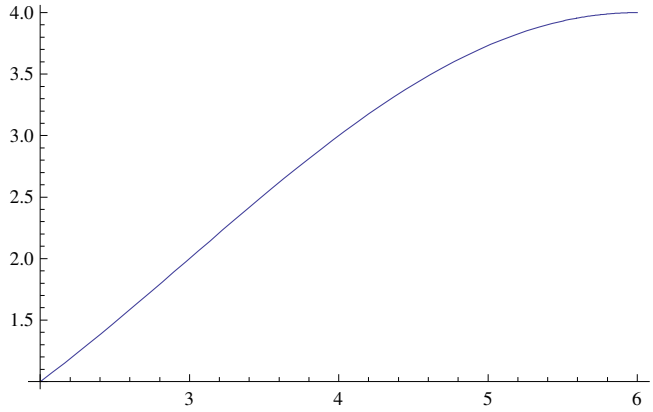
Out[8]=



In[9]:=
```
f[x_] := 4 Sin[Pi x / 12]^2
```

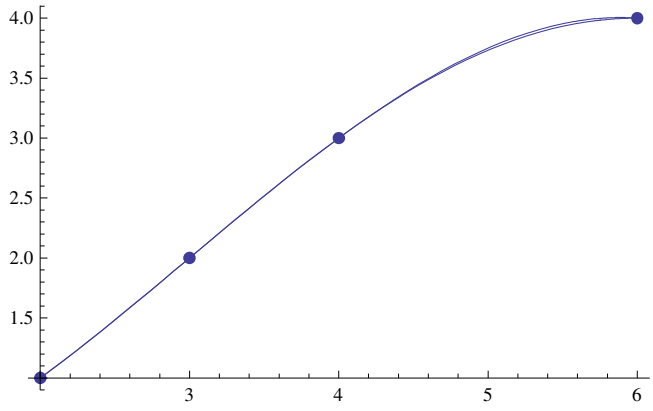
In[10]:=
```
a3 = Plot[f[x], {x, 2, 6}]
```

Out[10]=



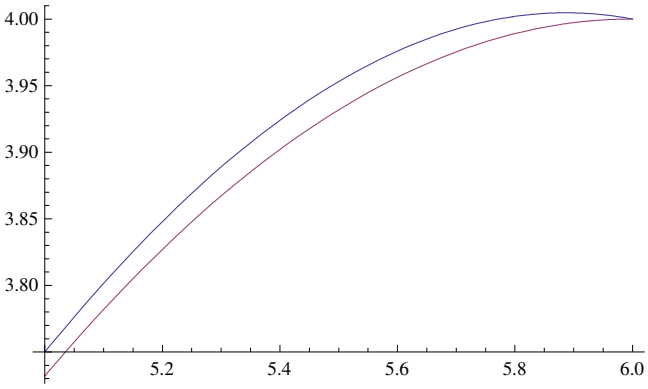
In[11]:=
```
Show[a1, a2, a3]
```

Out[11]=



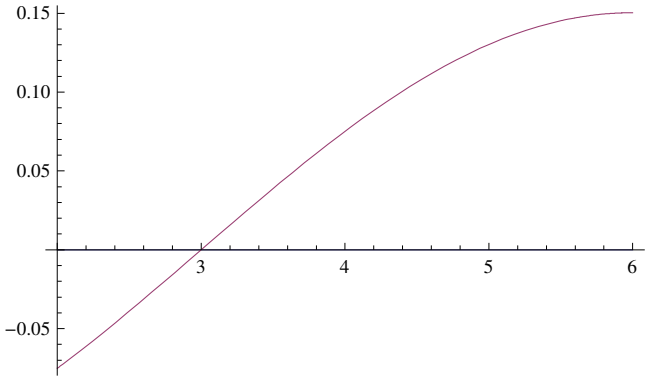
In[12]:=
```
Plot[{p[x], f[x]}, {x, 5, 6}]
```

Out[12]=



In[13]:=
```
Plot[{0, f''''[x]}, {x, 2, 6}]
```

Out[13]=



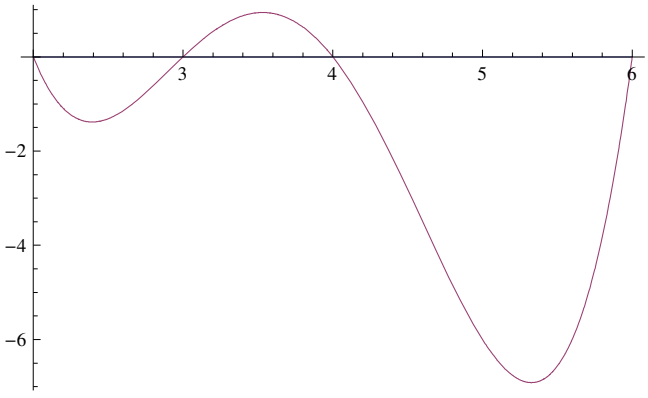
In[14]:=
```
M1 = FindMaximum[f''''[x], {x, 5}]
```

Out[14]=
```
{0.150323, {x → 6.}}
```

In[15]:=
```
w[x_] := (x - 2) (x - 3) (x - 4) (x - 6)
```

In[16]:=
```
Plot[{0, w[x]}, {x, 2, 6}]
```

Out[16]=



In[17]:=
```
M2 = FindMaximum[-w[x], {x, 5}]
```

Out[17]=
```
{6.9141, {x → 5.32635}}
```

In[18]:=
```
M1[[1]] M2[[1]] / 4!
```

Out[18]=
```
0.0433061
```

In[19]:=
```
FindMaximum[Abs[f[x] - p[x]], {x, 5}]
```

FindMaximum::lstol :
The line search decreased the step size to within tolerance specified by AccuracyGoal and PrecisionGoal but was unable to find a sufficient increase in the function. You may need more than MachinePrecision digits of working precision to meet these tolerances. ≫

Out[19]=
```
{0.0219112, {x → 5.3747}}
```

```
{2, 1}, {3, 2}, {4, 3}, {6, 4}
```

In[1]:=
```
A[n_] := {Subscript[a, n], Subscript[b, n], Subscript[c, n], Subscript[d, n]}
```

In[2]:=
```
p[n_, m_, x_] := D[A[n].{t^3, t^2, t, 1}, {t, m}] /. t → x
```

In[3]:=
```
B = {p[0, 0, 2] == 1, p[0, 0, 3] == 2,
   p[1, 0, 3] == 2, p[1, 0, 4] == 3,
   p[2, 0, 4] == 3, p[2, 0, 6] == 4,
   p[0, 1, 3] == p[1, 1, 3],
   p[1, 1, 4] == p[2, 1, 4],
   p[0, 2, 3] == p[1, 2, 3],
   p[1, 2, 4] == p[2, 2, 4],
   p[0, 2, 2] == 0, p[2, 2, 6] == 0}
```

Out[3]=
$$\{8\,a_0 + 4\,b_0 + 2\,c_0 + d_0 == 1,\ 27\,a_0 + 9\,b_0 + 3\,c_0 + d_0 == 2,\ 27\,a_1 + 9\,b_1 + 3\,c_1 + d_1 == 2,$$
$$64\,a_1 + 16\,b_1 + 4\,c_1 + d_1 == 3,\ 64\,a_2 + 16\,b_2 + 4\,c_2 + d_2 == 3,\ 216\,a_2 + 36\,b_2 + 6\,c_2 + d_2 == 4,$$
$$27\,a_0 + 6\,b_0 + c_0 == 27\,a_1 + 6\,b_1 + c_1,\ 48\,a_1 + 8\,b_1 + c_1 == 48\,a_2 + 8\,b_2 + c_2,$$
$$18\,a_0 + 2\,b_0 == 18\,a_1 + 2\,b_1,\ 24\,a_1 + 2\,b_1 == 24\,a_2 + 2\,b_2,\ 12\,a_0 + 2\,b_0 == 0,\ 36\,a_2 + 2\,b_2 == 0\}$$

In[4]:=
```
B // TableForm
```

Out[4]//TableForm=
$$8\,a_0 + 4\,b_0 + 2\,c_0 + d_0 == 1$$
$$27\,a_0 + 9\,b_0 + 3\,c_0 + d_0 == 2$$
$$27\,a_1 + 9\,b_1 + 3\,c_1 + d_1 == 2$$
$$64\,a_1 + 16\,b_1 + 4\,c_1 + d_1 == 3$$
$$64\,a_2 + 16\,b_2 + 4\,c_2 + d_2 == 3$$
$$216\,a_2 + 36\,b_2 + 6\,c_2 + d_2 == 4$$
$$27\,a_0 + 6\,b_0 + c_0 == 27\,a_1 + 6\,b_1 + c_1$$
$$48\,a_1 + 8\,b_1 + c_1 == 48\,a_2 + 8\,b_2 + c_2$$
$$18\,a_0 + 2\,b_0 == 18\,a_1 + 2\,b_1$$
$$24\,a_1 + 2\,b_1 == 24\,a_2 + 2\,b_2$$
$$12\,a_0 + 2\,b_0 == 0$$
$$36\,a_2 + 2\,b_2 == 0$$

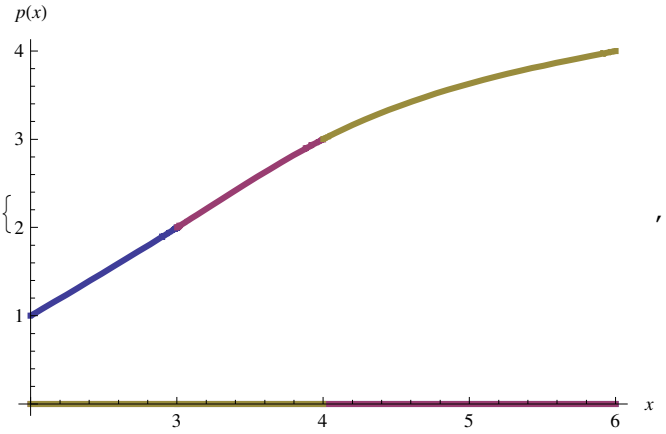
In[5]:=
```
T = Solve[B, Union[A[0], A[1], A[2]]]
```

Out[5]=
$$\left\{\left\{a_0 \to \frac{1}{46},\ a_1 \to -\frac{5}{46},\ a_2 \to \frac{1}{23},\ b_0 \to -\frac{3}{23},\ b_1 \to \frac{24}{23},\right.\right.$$
$$\left.\left. b_2 \to -\frac{18}{23},\ c_0 \to \frac{57}{46},\ c_1 \to -\frac{105}{46},\ c_2 \to \frac{231}{46},\ d_0 \to -\frac{26}{23},\ d_1 \to \frac{55}{23},\ d_2 \to -\frac{169}{23}\right\}\right\}$$

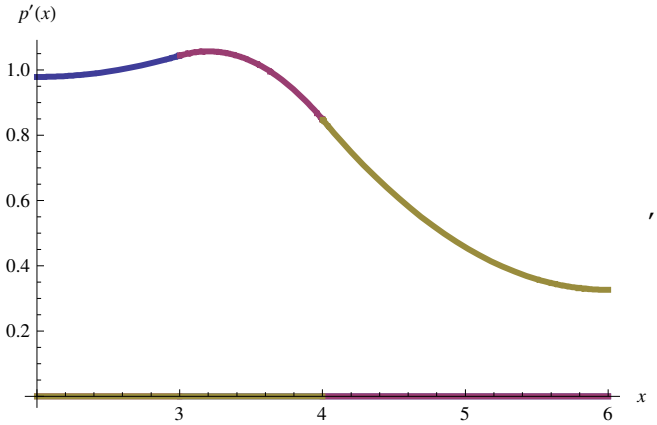
In[6]:=
```
Do[A[k] = A[k] /. T[[1]], {k, 0, 2}]
```

In[7]:=
```
Table[Plot[{p[0, m, x] UnitStep[-x + 3], p[1, m, x] UnitStep[x - 3] UnitStep[-x + 4],
    p[2, m, x] UnitStep[x - 4]}, {x, 2, 6}, PlotStyle → Thickness[0.01],
  PlotRange → All, AxesLabel → {x, D[p[x], {x, m}]}], {m, 0, 3}]
```

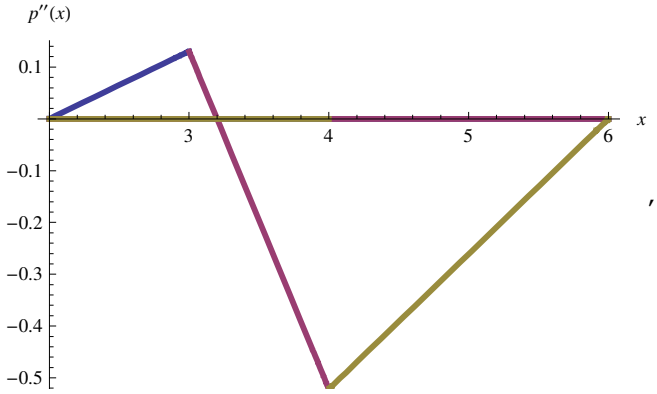
Out[7]= {



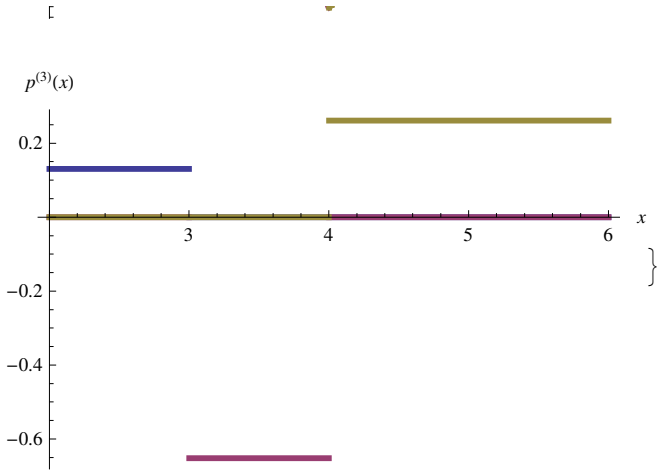
,



,



,


$p^{(3)}(x)$
0.2
−0.2
−0.4
−0.6
3
4
5
6
x

In[1]:=
```
g[n_, m_] := ∫_-1^1 x^n dx - Sum[Subscript[w, k] Subscript[x, k]^n, {k, 1, m}]
```

In[2]:=
```
B = Table[g[n, 3] == 0, {n, 0, 5}]
```

Out[2]=
$$\left\{2 - w_1 - w_2 - w_3 == 0,\ -w_1 x_1 - w_2 x_2 - w_3 x_3 == 0,\ \frac{2}{3} - w_1 x_1^2 - w_2 x_2^2 - w_3 x_3^2 == 0,\right.$$
$$\left. -w_1 x_1^3 - w_2 x_2^3 - w_3 x_3^3 == 0,\ \frac{2}{5} - w_1 x_1^4 - w_2 x_2^4 - w_3 x_3^4 == 0,\ -w_1 x_1^5 - w_2 x_2^5 - w_3 x_3^5 == 0\right\}$$

In[3]:=
```
B // TableForm
```

Out[3]//TableForm=
$$2 - w_1 - w_2 - w_3 == 0$$
$$-w_1 x_1 - w_2 x_2 - w_3 x_3 == 0$$
$$\frac{2}{3} - w_1 x_1^2 - w_2 x_2^2 - w_3 x_3^2 == 0$$
$$-w_1 x_1^3 - w_2 x_2^3 - w_3 x_3^3 == 0$$
$$\frac{2}{5} - w_1 x_1^4 - w_2 x_2^4 - w_3 x_3^4 == 0$$
$$-w_1 x_1^5 - w_2 x_2^5 - w_3 x_3^5 == 0$$

In[4]:=
```
T = Flatten[Table[{Subscript[w, k], Subscript[x, k]}, {k, 1, 3}]]
```

Out[4]=
$$\{w_1, x_1, w_2, x_2, w_3, x_3\}$$

In[5]:=
```
Solve[B, T]
```

Out[5]=
$$\left\{\left\{w_1 \to \frac{5}{9}, w_2 \to \frac{5}{9}, w_3 \to \frac{8}{9}, x_3 \to 0, x_2 \to -\sqrt{\frac{3}{5}}, x_1 \to \sqrt{\frac{3}{5}}\right\},\right.$$
$$\left\{w_1 \to \frac{5}{9}, w_2 \to \frac{5}{9}, w_3 \to \frac{8}{9}, x_3 \to 0, x_2 \to \sqrt{\frac{3}{5}}, x_1 \to -\sqrt{\frac{3}{5}}\right\},$$
$$\left\{w_1 \to \frac{5}{9}, w_2 \to \frac{8}{9}, w_3 \to \frac{5}{9}, x_3 \to -\sqrt{\frac{3}{5}}, x_2 \to 0, x_1 \to \sqrt{\frac{3}{5}}\right\},$$
$$\left\{w_1 \to \frac{5}{9}, w_2 \to \frac{8}{9}, w_3 \to \frac{5}{9}, x_3 \to \sqrt{\frac{3}{5}}, x_2 \to 0, x_1 \to -\sqrt{\frac{3}{5}}\right\},$$
$$\left\{w_1 \to \frac{8}{9}, w_2 \to \frac{5}{9}, w_3 \to \frac{5}{9}, x_3 \to -\sqrt{\frac{3}{5}}, x_2 \to \sqrt{\frac{3}{5}}, x_1 \to 0\right\},$$
$$\left.\left\{w_1 \to \frac{8}{9}, w_2 \to \frac{5}{9}, w_3 \to \frac{5}{9}, x_3 \to \sqrt{\frac{3}{5}}, x_2 \to -\sqrt{\frac{3}{5}}, x_1 \to 0\right\}\right\}$$

In[6]:=
```
f[x_] := Exp[-x^2 / 2]
```

In[7]:=
$$\frac{5}{9}\,f\left[-\sqrt{\frac{3}{5}}\,\right]+\frac{8}{9}\,f[0]+\frac{5}{9}\,f\left[\sqrt{\frac{3}{5}}\,\right]$$

Out[7]=
$$\frac{8}{9}+\frac{10}{9\,e^{3/10}}$$

In[8]:=
```
N[%]
```

Out[8]=
```
1.71202
```

In[9]:=
$$N\left[\int_{-1}^{1} f[x]\,dx\right]$$

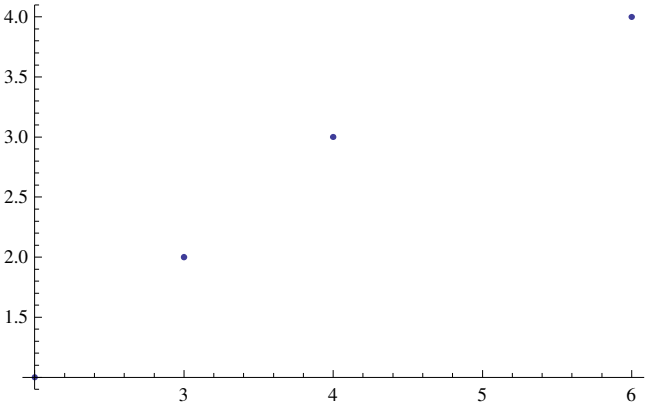
Out[9]=
```
1.71125
```

In[1]:=
```
A = {{2, 1}, {3, 2}, {4, 3}, {6, 4}}
```

Out[1]=
```
{{2, 1}, {3, 2}, {4, 3}, {6, 4}}
```

In[2]:=
```
a0 = ListPlot[A, PlotStyle → PointSize[0.01]]
```

Out[2]=



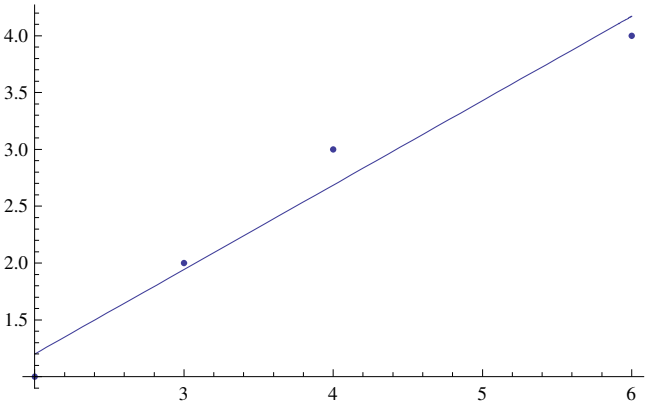
In[3]:=
```
p1[x_] = Fit[A, {1, x}, x]
```

Out[3]=
```
-0.285714 + 0.742857 x
```

In[4]:=
```
a1 = Plot[p1[x], {x, 2, 6}];
```

In[5]:=
```
Show[a0, a1]
```

Out[5]=



In[6]:=
```
X = Table[A[[n]][[1]], {n, 1, 4}]
```

Out[6]=
```
{2, 3, 4, 6}
```

In[7]:=
```
Y = Table[A[[n]][[2]], {n, 1, 4}]
```

Out[7]=
```
{1, 2, 3, 4}
```

In[8]:=
```
Correlation[X, Y]
```

Out[8]=
$$\frac{13}{5\sqrt{7}}$$

In[9]:=
```
N[%]
```

Out[9]=
```
0.982708
```

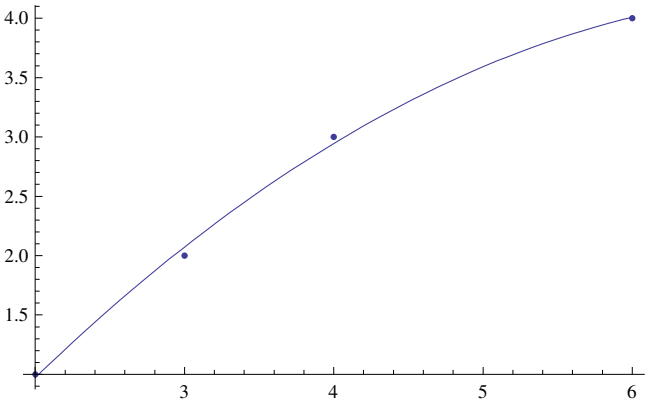
In[10]:=
```
p2[x_] = Fit[A, {1, x, x^2}, x]
```

Out[10]=
$$-1.90909 + 1.66818\,x - 0.113636\,x^2$$

In[11]:=
```
a2 = Plot[p2[x], {x, 2, 6}];
```

In[12]:=
```
Show[a0, a2]
```

Out[12]=



In[13]:=
```
p3[x_] = Fit[A, {E^x, Sin[x], Cos[x]}, x]
```

Out[13]=
$$0.0149956\,e^x - 2.22907\,\text{Cos}[x] - 0.454109\,\text{Sin}[x]$$

In[14]:=
```
a3 = Plot[p3[x], {x, 2, 6}];
```

In[15]:=
```
Show[a0, a3]
```

Out[15]=