

One method of doing numerical integration is Gaussian Quadrature. Note that both the Trapezoidal and the Simpsons rules looks like $\int_a^b f(x)dx \approx \sum_k w_k f(x_k)$ and we knew x_k and found w_k . In this method we find both x_k and w_k so that the integral and the sum are equal for a given n degree polynomial $p(x)$. It is achieved by forcing both sides equal for each power of x^j for $j = 0, 1, 2, \dots, n$. What is the degree of the polynomial we need to use if we want 3 points and the corresponding 3 weights? Find them for $[a, b] = [-1, 1]$ and use it to approximate $\int_0^1 \sin(x^2)dx$.