

For the data set  $\{(x_k, y_k)\}, k = 0, 1, \dots, n$  a natural cubic spline is a twice differentiable piecewise cubic polynomial  $p(x)$  which satisfies  $p(x_k) = y_k$  with  $p''(x_0) = p''(x_n) = 0$ . Let  $p(x) = \sum_{k=0}^{n-1} p_k(x)$  where  $p_k(x)$  is the part of  $p(x)$  on  $[x_k, x_{k+1}]$  which is 0 elsewhere. Assume that  $p_k(x) = a_k(x - x_k)^3 + b_k(x - x_k)^2 + c_k(x - x_k) + d_k$  and that  $x_{k+1} - x_k = h$  is a constant. With  $s_k = p''(x_k)$  derive the formula  $s_{k+2} + 4s_{k+1} + s_k = \frac{6}{h^2}(y_{k+2} - 2y_{k+1} + y_k), k = 0, 1, \dots, n - 2$ . Also write the system of equations in matrix form that must be solved to find  $s_k$  for  $k = 1, \dots, n - 1$ . Get the value of USD w.r.t. LKR for the 1st day of every month this year and predict the value for the 1st November this year using natural cubic splines (just show the data set, the calculated  $s_k$  values and the final answer).