Q1. Find $\ln(1.5)$ accurate to 10^{-6} using the integral from of the Taylor series remainder.

Q2. Prove that the Taylor series remainder has the integral from $R_n(x,a) = \frac{1}{n!}\int_a^x f^{(n+1)}(t)(x-t)^n dt$ for $f \in \mathcal{C}^{n+1}$ and use it to find $\ln(0.2)$ accurate to 10^{-6}.

Q3. Use the integral from of the Taylor series remainder $R_n(x,a) = \frac{1}{n!}\int_a^x f^{(n+1)}(t)(x-t)^n dt$ for to derive its differential form $R_n(x,a) = \frac{f^{(n+1)}(\zeta)}{(n+1)!}(x-a)^{n+1}$ where ζ is between x and a provided $f \in \mathcal{C}^{n+1}$

Q4. Let $f(x) = \begin{cases} 2 \text{ if } x \text{ is a rational number on } [a,b] \\ 1 \text{ if } x \text{ is an irrational number on } [a,b] \end{cases}$.
Show that $f \notin \mathcal{R}[a,b]$ for $b > a$

Q5. Prove the following with suitable conditions
1. Integration by parts: $\int_a^b f(x)g'(x)dx = f(b)g(b) - f(a)g(a) - \int_a^b g(x)f'(x)dx$
2. Change of variable: $\int_{g(a)}^{g(b)} f(y)dy = \int_a^b f(g(x))g'(x)dx$.
3. Leibniz rule: $\frac{d}{dx}\int_{a(x)}^{b(x)} f(t)dt = f(b(x))b'(x) - f(a(x))a'(x)$

Q6.
1. Find the range of x such that $\int_{0+}^1 e^{-t}t^{x-1}dt$ converges and conclude that $\Gamma(x)$ exists iff $x > 0$.
2. Show that $\lim_{x\to\infty}\Gamma(x) = \infty = \lim_{x\to 0^+}\Gamma(x)$

Q7. Find $\int_0^1 e^{-x^2}dx$ accurate to 10^-3 using the trapezoidal rule.

Q8. Find $\int_0^1 e^{-x^2}dx$ accurate to 10^-3 using the Simpson's rule.

Q9. Find the number of equispaced points needed to approximate $f(x) = 4\sin^2(\frac{\pi x}{12})$ on $[2,6]$ accurate to 10^{-4}.

Q10. Use the Newton's method formula and analysis based of the fixed point method to find the root of $f(x) = x - e^{-x}$ accurate to 10^{-4} on $[0,1]$.

Q11. Prove the Mean Value Theorem
1. $f : \mathbb{D} \to \mathbb{R}, \mathbb{D} = \{(x,y)|(x-a)^2 + (y-b)^2 < \delta^2\}$
2. f_x and f_y exists on $\mathbb{D}$
3. $\Delta x^2 + \Delta y^2 < \delta^2$
Then
1. $f(a+\Delta x, b+\Delta y) = f(a,b) + \Delta x f_x(a+\theta\Delta x, b) + \Delta y f_y(a+\Delta x, b+\alpha\Delta y)$
and
$f(a+\Delta x, b+\Delta y) = f(a,b) + \Delta y f_y(a, b+\beta\Delta y) + \Delta x f_x(a+\gamma\Delta x, b+\Delta y)$
2. $0 < \theta, \alpha, \beta, \gamma < 1$

Q12. Find the maximum and minimum directional derivatives of $f(x, y) = xy^2 + \sin x + \log y$ at $(0, 1)$.

Q13. Fit a least square line for the data set $\{(2, 1), (3, 2), (4, 3), (6, 4)\}$.