B1. Let $f(x)=\frac{x^2}{e^x-1}$ for $x\neq 0$. Use the variable change $t=e^x-1$ and the Taylor series $\ln(1+t)=\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{k}t^k=t-\frac{1}{2}t^2+\frac{1}{3}t^3-\cdots\cdots, |t|<1$ to show that $f(x)=t-t^2+\frac{11}{12}t^3+\cdots\cdots=x-\frac{1}{2}x^2+\cdots\cdots$ for x close to 0.

Can we define $f(0)$ so that it is continuous at 0? If so find such a $f(0)$ and deduce $f'(0), f''(0)$. State any assumptions.

B2. Show that $\int_0^\infty \frac{x^2}{e^x-1}\,dx$ is converging. State any assumptions.

C1. Write $\int_0^{\infty} \frac{x^2}{e^x-1} dx = \int_0^{m} \frac{x^2}{e^x-1} dx + \int_m^{\infty} \frac{x^2}{e^x-1} dx$. Find an upper bound for the second integral $\int_m^{\infty} \frac{x^2}{e^x-1} dx$ that is approaching 0 as $m \to \infty$. Hence find and integer m such that $\int_m^{\infty} \frac{x^2}{e^x-1} dx < \frac{10^{-4}}{2}$.

C2. Assume we got $m = 23$ for the answer in C1. Use the error in Trapezoidal rule to find the number of divisions n needed to find $\int_0^{23} \frac{x^2}{e^x-1} dx$ accurate to $\frac{10^{-4}}{2}$ assuming $|f''(x)| \leq 1$. Do not find this Trapeziodal approximation, but show that it will give us the value of $\int_0^{\infty} \frac{x^2}{e^x-1} dx$ accurate to 10^{-4}.