FINANCIAL MATHEMATICS

Instructions: Answer five questions only.

Time: Three (03) hours.

Each question carries equal marks

Q1. (a) Explain briefly three types of traders in the stock market.

An investor would like to speculate on a rise in the price of a certain stock. The current stock price is Rs. 29 and a three month European call with a strike price of Rs. 30, cost Rs. 2.90. Suppose the investor has Rs. 5800 to invest. Identify two alternative strategies and briefly outline the advantages and disadvantages of each.

- (b) A six month American put option on a stock is expected to pay dividends of \$2.06, per share at the end of the 3.5 months. The current stock price is \$30, the exercise price is \$34, the risk free interest rate is 10% per annum and the volatility of the part of the stock price that will not be used to play dividends is 30% per annum. Divide the life of option into three, two month periods and use the binomial tree approach to estimate the value of the option. Give your answer up to second decimal place.
- Q2. (a) An investor has just obtained the following quotes for three month options on a stock worth \$29, Strike price is \$30, Value of European call option \$3, risk free interest rate 10% p.a. and Put option value is \$1. Identify the arbitrage opportunity and explain it, using cash flow diagram. What is the value of European call option for no arbitrage if value of others is same?
 - (b) Three call options on a stock have same expiration date and strike prices of \$ 55, \$60 and \$65: the call prices are \$3, \$5 and \$8 respectively. Explain how a butterfly spread can be created. Construct a figure showing the profit from the strategy. For what range of stock prices would the butterfly spread lead to a loss?
- Q3. A fund manager has a portfolio worth Rs. 50 million, with beta of 0.82. The manager is concern about the performance of the market over the next two months and plans to use three month futures contract on the S&P 100 to hedge the risk. The current level of index is Rs. 1250, one contract is on 100 times the index, the risk free rate is 6% per annum and the dividends yield on the index is 3% per annum

- (a) What is the theoretical future price for the three month futures contract?
- (b) What position should be fund manager take to eliminate all exposure to the market over the next two months?
- (c) Calculate the effect of your strategy on the fund manager's returns if the level of the market in the two months is 1000 and 2000.
- Q4. (a) Show that a standard Brownian motion (BM) $\{B(t), t \ge 0\}$ is a Martingale with respect to natural filtration.
 - (b) Let $\{ M_n \mid n \ge 0 \}$ and $\{ N_n \mid n \ge 0 \}$ are martingale with $E(M_n^2) < \infty$ and $E(N_n^2) < \infty$ for all *n*. Show that

 $E(M_n N_n) - E(M_0 N_0) = \sum_{m=1}^{n} E((M_m - M_{m-1})(N_m - N_{m-1})).$

(c) If D_1 is a sub σ - algebra of D_2 then show that

 $E \ (\ E \ (X \mid D_2) \mid D_1 \) \ = \ E \ (\ E \ (X \mid D_1) \mid D_2 \) = \ E \ (\ X \mid D_1 \).$

Q5. (a) Suppose that the stock price $\{S(t), t \ge 0\}$ satisfies the stochastic differential equation

 $dS(t) = \mu S(t) dt + \sigma S(t) dB(t)$ Where $\{B(t), t \ge 0\}$ is a Brownian motion.

Derive the Black-Sholes- Marton differential equation for f, the price of a derivative on the stock.

- (b) What is the price of a European call option on a non-dividend paying stock when the stock price is \$50; the risk free interest rate is 12% per annum. The volatility is 30% per annum and the time of maturity is three months.
- Q6. (a) What is risk natural measure?

Under the risk natural measure Q,

$$c(0) = e^{-rT} E_0 [(S(T) - K)^+]$$

All the terms are in usual notations and

$$S(T) = S(0) \left[\exp\left\{ \left(\mu - \frac{\sigma^2}{2} \right) T + \sigma B(T) \right\} \right]$$

Show that

$$c(0) = S(0) \varphi (d_1) - K e^{-rT} \varphi (d_2)$$

Where $d_1 = \frac{\ln (\frac{S(0)}{K}) + rT + \frac{1}{2} \sigma^2 T}{\sigma \sqrt{T}}$
 $d_2 = d_1 - \sigma \sqrt{T}$.

(b) State clearly, assumptions under Black Scholers pricing model.

- Q7. (a) Explain the difference between a future contract on a stock and an option on a stock.
 - (b) A stock index currently stands at 300. The risk free interest rate is 12% p.a. and the dividend yield on the index is 6% p. a. What should the future price for a four- month contract be?

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- (c) Explain how the European call and put option prices change with stock price, strike price and time of expiration.
- (d) Suppose that F_1 and F_2 are two futures contracts on the same commodity with times to maturity t_1 and t_2 . Prove that $F_2 \le F_1 \exp(r(t_1 t_2))$ where r is the interest rare (assumed constant) and there are no stoppage costs.

Date of Examination: 19.10.2008