Example 4

1. The breaking strains of reels of string produced at a certain factory have a standard deviation of 1.5 kg. A sample of 100 reels from a certain batch were tested and their mean breaking strain was 5.30 kg.
(a) Find a 95% confidence interval for the mean breaking strain of string in this batch.
(b) The manufacturer becomes concerned if the lower 95% confidence limit falls below 5 kg. A sample of 80 reels from another batch gave a mean breaking strain of 5.31 kg. Will the manufacturer be concerned?

2. A random sample of size 25 is taken from a normal population with standard deviation of 2.5. The mean of the sample was 17.8.
(a) Find a 99% confidence interval for the population mean μ.
(b) What size sample is required to obtain a 99% confidence interval of width at most 1.5?
(c) What confidence level would be associated with the interval based on the above sample of 25 but of width 1.5, i.e. (17.05, 18.55)?

3. At a certain college new students are weighed when they join the college. The distribution of weights of students at the college when they enroll is normal, with a standard deviation of 7.5 kg and a mean of 70 kg. A random sample of 90 students from the new entry were weighed and their mean weight was 71.6 kg. Assuming that the standard deviation has not changed and that the weights of the new class were also normally distributed, test, at the 5% level, whether or not there is evidence that the mean of the new entry is more than 70kg.

4. A machine produces bolts of diameter D, where D has a normal distribution with mean 0.580cm and standard deviation 0.015 cm. The machine is serviced and after the service a random sample of 50 bolts from the next production run is taken to see if the mean diameter of the bolts has changed from 0.580cm. The distribution of the diameters of bolts after the service is still normal with a standard deviation of 0.015cm. The mean diameter of the 50 bolts is 0.577cm. Test, at the 1% level, whether or not there is evidence that the mean diameter of the bolts has changed.