MATRICES

Exercises 3

1. Given $A = \begin{pmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{pmatrix}$ $B = \begin{pmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{pmatrix}$ and $C = \begin{pmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{pmatrix}$

(a) Compute that A+B and A-C

(b) Verify that A+(B+C) = (A+B)+C

(c) Compute AB, BA and AC^T

Q(2). Evaluate :

$$\begin{pmatrix} 0 & 1 & 3 \\ 4 & 5 & 6 \\ 7 & 3 & 2 \end{pmatrix}\begin{pmatrix} 1 & 4 & 5 \\ 6 & 3 & 2 \\ 2 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 12 & 12 & 5 \\ 46 & 49 & 36 \\ 29 & 43 & 43 \end{pmatrix}$$

Q(3). If $A = \begin{pmatrix} 1 & 2 \\ 4 & -3 \end{pmatrix}$, show that $A^2 = \begin{pmatrix} 9 & -4 \\ -8 & 17 \end{pmatrix}$ and $A^3 = \begin{pmatrix} -7 & 30 \\ 60 & -67 \end{pmatrix}$

Q(4). If $A = \begin{pmatrix} -1 & -2 & -2 \\ 1 & 2 & 1 \\ 1 & -1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 2 & 2 \\ -1 & 0 & -1 \\ -3 & -3 & -2 \end{pmatrix}$

Show that $\{kA + (1-k)B\}^2 = I$, k being a scalar.

Q(5). Find x, y such that $\begin{pmatrix} 2 & -1 \\ -3 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} 8 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

Q(6). Express the matrix $\begin{pmatrix} 1 & 2 & 4 \\ 2 & 3 & -1 \\ -3 & 1 & 4 \end{pmatrix}$ as the sum of symmetric and skew symmetric matrix.

$$S = \begin{pmatrix} 1 & 2 & 1/2 \\ 2 & 3 & 0 \\ 1/2 & 0 & 4 \end{pmatrix} \text{ and } T = \begin{pmatrix} & 0 & 7/2 \\ 0 & 0 & -1 \\ -7/2 & 1 & 0 \end{pmatrix}$$

Exercises 3.5

Q(1). If $A = \begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$, show that $A^3 = A^{-1}$.

Q(2). Find the Inverse of (a) $\begin{pmatrix} 3 & 2 & 2 \\ 2 & 5 & 3 \\ 1 & 2 & 1 \end{pmatrix}$,

(b) $A = \begin{pmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{pmatrix}$

Use elementary row operations

Find the inverse of the matrix $\begin{pmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix}$ by elementary row transformations.

Q(5). If $A = \begin{pmatrix} 1 & -a & 1 \\ b & 0 & 2b \\ 0 & a & 0 \end{pmatrix}$ then, show that

(i) $A^3 = abA + A^2 - abI$, is satisfied

(ii) Show also that $A^{2n} = \left\{ \frac{(ab)^n - 1}{ab - 1} \right\} A^2 - \left\{ \frac{ab\{(ab)^{n-1} - 1\}}{ab - 1} \right\} I$, where n is a positive

6.(a) Show that every 2x2 matrix such that $X^T A X = B,$ where

$A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},\quad B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ has one of the forms $\begin{pmatrix} a & \frac{1}{2a} \\ a & \frac{1}{-2a} \end{pmatrix}$ or $\begin{pmatrix} a & \frac{1}{2a} \\ -a & \frac{1}{2a} \end{pmatrix}$

(b) If $P = QRQ^{-1}$, show that $P^n = QR^nQ^{-1}$ where n is a positive integer.

Try $P^2 = (QRQ^{-1})(QRQ^{-1}) = QR^2Q^{-1}$

Similarly $P^3 = (QR^2Q^{-1})(QRQ^{-1}) = QR^3Q^{-1}$

In general $\underline{\underline{P^n = QR^nQ^{-1}}}$

(c) Let $P = \begin{pmatrix} 5 & -2 \\ 7 & -4 \end{pmatrix}$ and $Q = \begin{pmatrix} 2 & 1 \\ 7 & 1 \end{pmatrix}$, show that $Q^{-1}PQ = \begin{pmatrix} -2 & 0 \\ 0 & 3 \end{pmatrix}$. Hence

find P^n.